Transient elastography in anisotropic medium: Application to the measurement of slow and fast shear wave speeds in muscles

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From the measurement of a low frequency (50–150 Hz) shear wave speed, transient elastography evaluates the Young's modulus in isotropic soft tissues. In this paper, it is shown that a rod source can generate a low frequency polarized shear strain waves. Consequently this technique allows to study anisotropic medium such as muscle. The evidence of the polarization of low frequency shear strain waves is supported by both numeric simulations and experiments. The numeric simulations are based on theoretical Green's functions in isotropic and anisotropic media (hexagonal system). The experiments *in vitro* led on beef muscle proves the pertinent of this simple anisotropic pattern. Results *in vivo* on man biceps shows the existence of slow and fast shear waves as predicted by theory. [DOI: 10.1121/1.1579008]

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I. INTRODUCTION

Anisotropy has long been studied in geophysics[1,2] or in nondestructive testing with ultrasound.[3,4] In biological media *ex vivo*, Hoffmeister[5] and Kuo[6] on tendon, Yoon[7] on bone or Levinson[8] and Andersen[9] on muscle quantify the anisotropic elastic moduli. Now elastographic techniques, using the propagation of a low frequency shear wave[10,11] can measure the Young's modulus of soft tissues. In this paper, we describe how these methods can be adapted to study the anisotropic shear moduli of simple muscles *in vitro* and *in vivo*. In the first part, the transient elastography technique[12,13] is presented. In the second part, theoretical considerations from a hexagonal anisotropic medium (transverse isotropic system) predict the existence of a slow and a fast shear wave. It is shown that the use of a rod as a low frequency wave source polarizes the shear strain wave on the first 50 mm and thus enables one to select either of the shear wave. These results follows numerical simulations based on theoretical Green's function in isotropic and transversely isotropic medium. In the third section we present some experimental results *ex vivo* on beef muscle and *in vivo* on human biceps.

II. TRANSIENT ELASTOGRAPHY

A. Experimental setup

An ultrasonic transducer (5 MHz) is applied at the surface of a homogeneous Agar-gelatin phantom (3% Agar and 5% gelatin) (Fig. 1). The transducer, a 7 mm diameter and a 35 mm focal depth, is set up on a vibrator (Brüel & Kjær, type 4810). An ultrasonic pulse echo system is used with a 2 kHz recurrence frequency. The ultrasonic signals are sampled at 50 MHz and stored using a 9-bit digitizer with 2 Mbytes memory. The low frequency pulse (50 to 150 Hz) is obtained with a conventional function generator. The basic idea is that the low frequency vibrations generated by a piston source (in this case the transducer itself) induce a low frequency motion of the scatterers inside the media. This motion can be detected and measured with a cross correlation algorithm on successive *A*-scans. Such a method gives the displacement along the axis of the ultrasound beam with an accuracy of 1 μm.

B. Displacement induced by an acoustic pulse

When a pulse is applied on a semi-infinite, isotropic and homogeneous solid, two kinds of waves are generated: a compressional and a shear waves. Typical speeds in soft tissues are, respectively, $V_p = 1500\ \mathrm{m\,s^{-1}}$ and $V_s = 5\ \mathrm{m\,s^{-1}}$. These speeds are connected to the elastic Lamé coefficients λ, μ in isotropic elastic media,

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \tag{1}$$

$$V_s = \sqrt{\frac{\mu}{\rho}}. \tag{2}$$

ρ is the density. In order to describe soft tissues elasticity, the usual parameter is the Young's modulus E,

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}. \tag{3}$$

In soft media λ is 10^6 times bigger than μ, thus a good approximation of E is

$$E \cong 3\mu. \tag{4}$$

Finally, we can write the Young's modulus as function of the shear wave speed

$$E = 3\rho V_s^2. \tag{5}$$

The measurement of the speed is deduced from the experimental displacements of Fig. 2 using a spectral analysis

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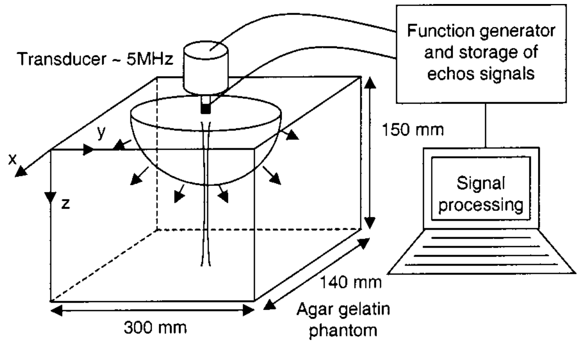

FIG. 1. Experimental setup. A 5 MHz ultrasonic transducer is set up on a vibrator. The acoustic impulse (100 Hz) is generated by the front face of the transducer while it works as a pulse echo system. The displacement field is calculated with a cross correlation technique on successive backscattered signals stored in memory.

at the central frequency. A linear fit on the phase as function of depth gives an accurate estimation of the phase velocity. The error is obtained from the standard deviation of the measurements from the linear fit. The values are $V_s = 2.88 \pm 0.03\ \mathrm{m\,s^{-1}}$ and using Eq. (5), $E = 8.29 \pm 0.14\ \mathrm{kPa}$ (the value of ρ is found in the literature to be $1100\ \mathrm{kg\,m^{-3}}$). Thus one can determine precisely the Young's modulus in isotropic media. Now if we consider anisotropic media such as muscles, what kind of elasticity does this technique give? To answer this question we will use the theory of anisotropic elastic media.

III. THEORY OF ELASTIC WAVES IN TRANVERSE ISOTROPIC SOLID

A. General case

The muscle model considered in this paper presents a random distribution of fibers oriented in the same direction Fig. 3. This consideration entails the existence of a symmetry axis along the fibers. It is proven[3] that this kind of symmetry corresponds to a hexagonal system (transverse isotropy). The Christoffel's matrix c_{ijkl} of such a system contains 5 independent elastic constants. The eigenvectors of the Christoffel's tensor (implied in the wave equation) are associated to eigenvalues which are the speeds of waves in all directions.

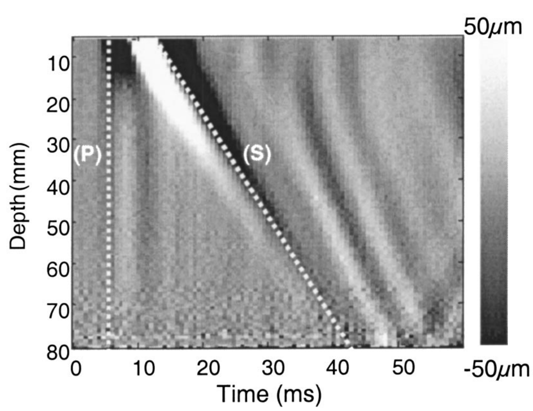

FIG. 2. Experimental displacement field obtained for an acoustic pulse (100 Hz). One can observe the displacements of a compressional wave (P) and of a shear wave (S) in an Agar-gelatin based phantom.

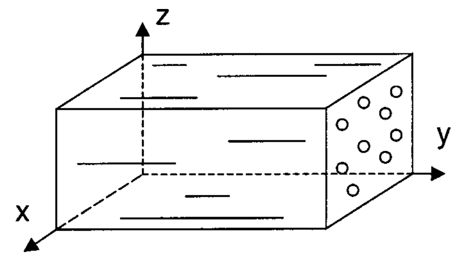

FIG. 3. Simple model of muscle. The fibers have the same orientation (along the y axis) and are randomly distributed on the plane (xz). The equivalent crystallographic model is the hexagonal system (transverse isotropy).

Nevertheless, we are only interested in wave propagation along the axis perpendicular to the muscle fibers (along x or z) since the direction parallel to the fibers is not easily accessible *in vivo* for the specific setup considered in this analysis. Moreover the weak backscattered signal for an ultrasonic beam parallel to this latter direction makes the use of transient elastography difficult. Therefore, in the direction perpendicular to the fibers (z direction) the speeds of the three different waves are

$$V_p = \sqrt{\frac{c_{11}}{\rho}}, \tag{6}$$

for the compressional wave,

$$V_s^{\perp} = \sqrt{\frac{c_{66}}{\rho}}, \tag{7}$$

for the shear wave with a polarization perpendicular to the fibers,

$$V_s^{\parallel} = \sqrt{\frac{c_{44}}{\rho}}, \tag{8}$$

for the shear wave with a polarization parallel to the fibers.

It clearly appears in Eqs. (7) and (8) that the elastic moduli c_{44} and c_{66} can be obtained if one is able to measure the speed of polarized shear waves. So how can one control the polarization of the low frequency shear wave along the axis of a piston source?

B. The polarization of shear strain waves

In the simple case of an isotropic medium, since the displacements preserve the symmetry of the initial boundary conditions, the shear strain field induced by a piston source is axial symmetric. Now the use of a rod source break this symmetry. Then the shear wave induces a shear strain field in a preferential direction (perpendicularly to the rod). We have first verify this hypothesis theoretically with the calculation of the exact Green's functions in an isotropic semi-infinite solid given by Gakenheimer and Miklowitz.[14] The parameters of the numerical simulation are $\rho = 1100\ \mathrm{kg\,m^{-3}}$, $V_p = 1500\ \mathrm{m\,s^{-1}}$, $V_s = 5\ \mathrm{m\,s^{-1}}$, a sampling frequency of 2 kHz, and a 80 mm rod length. Considering the rod as a 1 mm grid of point sources, we obtain the impulse response by summing the Green's function of all these individual point sources. This summation is the expression of the Rayleigh–Sommerfeld integral. Finally, a simple convolution of this

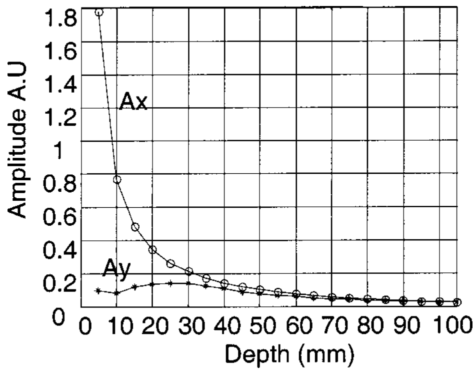

FIG. 4. Amplitude of the displacement gradient in the direction parallel (Ay) and perpendicular (Ax) to the rod versus depth. This simulation shows that the rod as a shear wave source favors the shear strain perpendicular to its main axis at least within the first centimeters of the isotropic solid model.

impulse response with the source excitation (a one-cycle sinusoid with a 100 Hz central frequency) gives the theoretical displacement field u_z. Figure 4 represents the amplitude of the gradient of the displacement (the shear strain field) in the direction parallel ($Ay=|\partial u_y/\partial y|$) and perpendicular ($Ax=|\partial u_x/\partial x|$) to the rod as function of depth. The axes are defined in Fig. 5. The strain field is smaller along the y axis than along the x axis until 40 mm depth. If we transpose this result to an anisotropic medium such as muscle, we can make the assumption that any speed measurement within this distance gives the speed of the shear wave with a polarization perpendicular to the fibers ($V_s^{\perp}$) when the rod is parallel to the fibers. Equally, if the rod is set perpendicular to the fibers, the speed of the shear wave with a polarization parallel to the fibers ($V_s^{\parallel}$) is obtained. We used a second numerical simulation in order to verify this latter assumption. The theoretical Green's function in an infinite transverse isotropic media are calculated by Vavryčuk[15] (see the Appendix). As shown by Sandrin,[16] Green's functions in isotropic infinite or semi-infinite soft media are very similar along the axis of the source. Thus the same assumption is made for anisotropic media. Again, by summing the Green's function from secondary point sources distributed on the surface of the rod, we get its impulse response. Then, the displacement field is computed for each angle of rotation of the rod as regard to the fibers from $\theta=0°$ to 180° with a 10° step (Fig. 6). For each angle, the shear wave speed (Fig. 7) is deduced from a spectral analysis of the simulated dis-

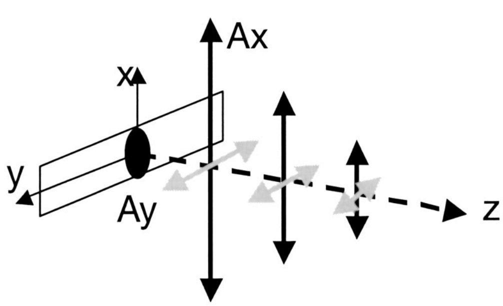

FIG. 5. 3D representation of the amplitude of the displacement gradient. The black and the gray arrows represent the gradient amplitude in the x and y direction, respectively.

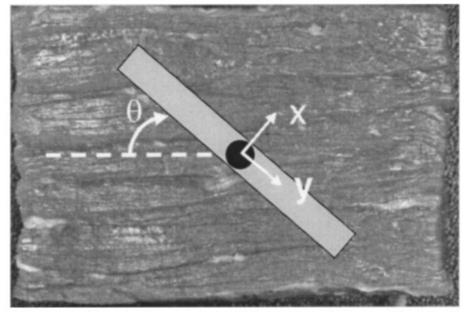

FIG. 6. Top view of an experimental setup. A rod (80 mm long) is used as a shear wave source. The speed is measured along the z axis for angle between the rod and the fibers ranging from $\theta=0°$ to 180°.

placements. The speed parameters used in this numerical simulation are chosen to be as close as possible to the experiment presented in the next section: $V_p=1500\ \mathrm{m\,s^{-1}}$, $V_s^{\perp}=10\ \mathrm{m\,s^{-1}}$, $V_s^{\parallel}=28\ \mathrm{m\,s^{-1}}$. The agreement between these values and the measured speeds is good especially for the two extreme positions (0° and 90°): they are, respectively, 29 $\mathrm{m\,s^{-1}}$ and 10 $\mathrm{m\,s^{-1}}$. The intermediate speeds are a combination of both shear speeds. Consequently these simulation results confirm the assumption that speed measurements with a rod as a shear wave source are very close to the values of polarized plane waves. The crucial point of this paragraph is that the use of a rod favors the strain field in the direction

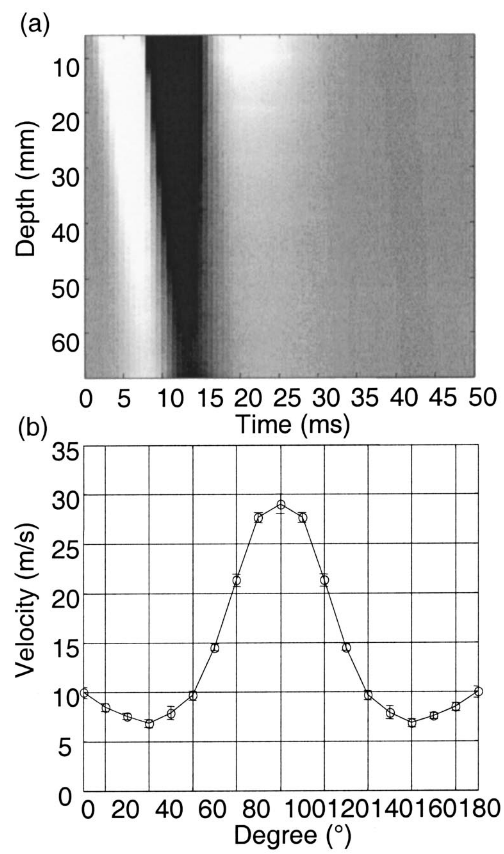

FIG. 7. An acoustic pulse given by a rod on an transverse isotropic media is simulated using the Vavryčuk's theoretical model. (a) Displacement field along depth versus time for $\theta=90°$ between the rod and the muscle fibers model. (b) The shear wave speed as function of the angle of rotation θ.

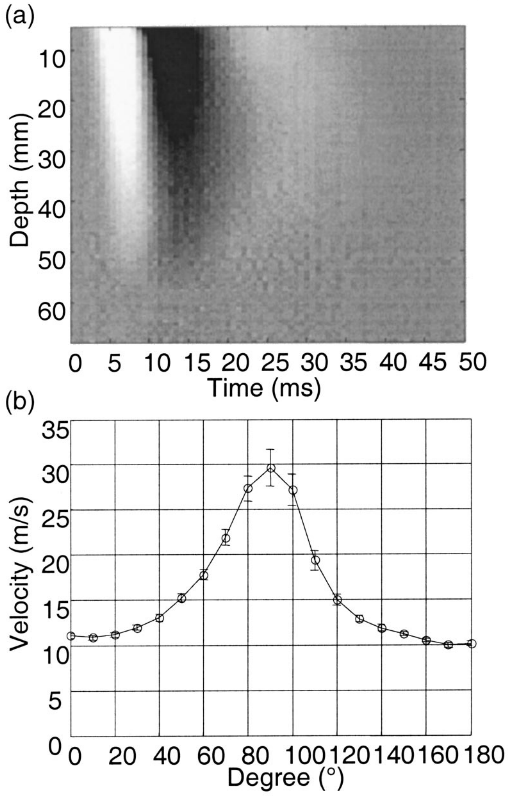

FIG. 8. An acoustic pulse is given by a rod on a beef muscle (*biceps femoris semi-tendinosus*). (a) Experimental displacement field along depth versus time for $\theta = 90°$ between the rod and the muscle fibers. (b) The shear wave speed as function of the angle of rotation θ.

perpendicular [x direction (Fig. 5)]. Nevertheless the polarization of the shear wave in the axis of the source remains longitudinal (z axis).

IV. EXPERIMENTAL RESULTS

A. *In vitro* experiment on beef muscle

In the experimental setup on beef muscle (*biceps femoris semi-tendinosus*), the fibers are clearly visible on the picture (Fig. 6) and well aligned. The ultrasonic equipment is described in the first part. The transducer is set in the middle of the rod (80 mm long) and allows to measure the longitudinal component of the displacement field along the z axis. A low frequency pulse at 100 Hz central frequency is generated with the rod applied on the surface of the sample. The speeds of the shear wave are measured for angles of rotation of the rod with regard to the fibers ranging from $\theta = 0°$ to $180°$ with a $10°$ step (as in the numerical experiment). The results (Fig. 8) show that the speed is maximum ($V_s^{\parallel} = 28\ \mathrm{m\,s^{-1}}$) when the rod is perpendicular to the fibers ($\theta = 90°$) and minimum ($V_s^{\perp} = 10\ \mathrm{m\,s^{-1}}$) when the rod is parallel to the fibers ($\theta = 0°$ and $\theta = 180°$). It is in good qualitative agreement with rheology studies on the same kind of muscles[17] since we find c_{66} (110 kPa), the shear elasticity perpendicular to the fibers, to be smaller than c_{44} (862 kPa), the shear elasticity parallel to the fibers. Moreover, the experimental results are in good quantitative agreement with the simulation (Fig. 7) although the viscosity has not been taken into account in our theoretical model.

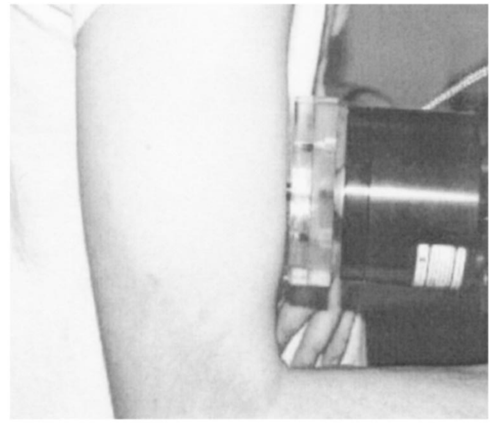

FIG. 9. Experimental setup on male biceps. The probe is applied on the surface of the biceps and the shear wave speed is measured.

B. *In vivo* experiment

The *in vivo* experiment presented here are done on the human biceps. One reason is that this muscle has a simple structure, the fibers are aligned in the same direction. A second reason is that it is often used as a reference in the literature. A volunteer lift at 6 kg charge. On the contracted muscle the rod system (Fig. 9) is applied. As in the previous experiments the shear wave speed is measured for various angles between the rod and the fibers. On the experimental curve of Fig. 10 the speed reaches a maximum when the rod is perpendicular to the fibers ($V_s^{\parallel} = 12\ \mathrm{m\,s^{-1}}$) and a minimum when the rod is parallel to the fibers ($V_s^{\perp} = 3\ \mathrm{m\,s^{-1}}$). These *in vivo* measurements are not surprising given the results of the simulation (Fig. 7) and of the *in vitro* experiments (Fig. 8). They confirm the idea that soft tissues like muscles are very anisotropic. As a comparison, in crystals or rocks, the ratio of the speed of waves in different direction of propagation rarely exceeds 2. It can be noted that the *in vivo* measurements are not as smooth as the *in vitro* experiment. A reason is that one cannot avoid tissue motions during the experiment. Moreover the speed is not measured on a long distance due to the finite size of the biceps (typically 2 cm) which increases the experimental error. Finally, in the case

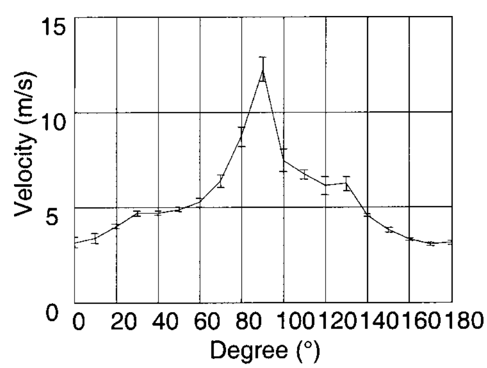

FIG. 10. *In vivo* measurements of the shear wave speed as function of the angle of rotation θ between the rod and the biceps fibers. A ratio of 4 is found between the fast and the slow shear wave which characterizes a strong anisotropy.

where the dimensions of the propagation medium are comparable to the shear wavelength, the boundary conditions may influence the measurements.

V. CONCLUSION

Since anisotropy in soft tissues is a rule more than an exception, elastographic techniques have to take into account this property in the measurement of the elasticity. We have shown in this paper that the use of a low frequency rod source in transient elastography enables one to study a simple transverse isotropic medium: the biceps. The validation of this technique is supported by numerical simulations based on Green's function in isotropic and hexagonal anisotropic solids (transverse isotropy). A strong anisotropy is found on the *in vitro* beef muscle ($V_s^{\perp}=10\ \mathrm{m\,s^{-1}}$, $V_s^{\parallel}=28\ \mathrm{m\,s^{-1}}$) and in the *in vivo* human biceps ($V_s^{\perp}=3\ \mathrm{m\,s^{-1}}$, $V_s^{\parallel}=12\ \mathrm{m\,s^{-1}}$). These results gave birth to a research program with the myology institute of the hospital ''La Pitié Salpetrière'' in Paris.

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APPENDIX: THE GREEN'S FUNCTION IN A HEXAGONAL ANISOTROPIC MEDIUM

In an hexagonal anisotropic, homogeneous and elastic infinite solid, the displacement field of a point source, the Green's function, is derived from higher-order ray theory (Vavryčuk). The solution is given in Cartesian coordinates $\mathbf{x}=(x,y,z)$:

$$G_{kl}(\mathbf{x},t)=\frac{1}{4\pi\rho}\{G_1(\mathbf{x},t)+G_2(\mathbf{x},t)+G_3(\mathbf{x},t)+G_4(\mathbf{x},t)+G_5(\mathbf{x},t)\}, \tag{A1}$$

where k and l are the direction index numbers of the displacement and the source, respectively. The other terms are defined as

$$G_1(\mathbf{x},t)\,\frac{1}{\sqrt{c_{11}^3}}\,\frac{g_{1k}g_{1l}}{\tau_1}\,\delta(t-\tau_1),$$

$$G_2(\mathbf{x},t)=\frac{1}{\sqrt{c_{44}^3}}\,\frac{g_{2k}g_{2l}}{\tau_2}\,\delta(t-\tau_2),$$

$$G_3(\mathbf{x},t)=\frac{1}{c_{66}\sqrt{c_{44}}}\,\frac{g_{3k}g_{3l}}{\tau_3}\,\delta(t-\tau_3), \tag{A2}$$

$$G_4(\mathbf{x},t)=\frac{1}{\sqrt{c_{44}}}\,\frac{g_{3k}^{\perp}g_{3l}^{\perp}-g_{3k}g_{3l}}{R^2}\int_{\tau_2}^{\tau_3}\delta(t-\tau)\,d\tau,$$

$$G_5(\mathbf{x},t)=\frac{3g_{1k}g_{1l}-\delta_{kl}}{r^3}\int_{\tau_1}^{\tau_2}r\,\delta(t-\tau)\,d\tau.$$

Here τ_1, τ_2, τ_3 are the travel time of, respectively, the compressional wave, the slow shear wave and the fast shear wave,

$$\tau_1=\frac{r}{\sqrt{c_{11}}}, \tag{A3}$$

$$\tau_2=\frac{r}{\sqrt{c_{44}}}, \tag{A4}$$

$$\tau_3=\frac{r}{\sqrt{c_{66}}}\sqrt{N_1^2+\frac{c_{66}}{c_{44}}N_3^2},\quad \tau_3=\frac{r}{\sqrt{c_{66}}}\sqrt{N_1^2+N_2^2+\frac{c_{66}}{c_{44}}N_3^2}. \tag{A5}$$

$R=\sqrt{x^2+y^2}$ is the distance to the receiver from the vertical axis, $r=\sqrt{x^2+y^2+z^2}$ is the distance from the source to the receiver. The polarization vectors are given by the following equations:

$$g_1=[N_1,N_2,N_3], \tag{A6}$$

$$g_2=\frac{-1}{\sqrt{N_1^2+N_2^2}}[-N_1N_3,-N_2N_3,N_1^2+N_2^2], \tag{A7}$$

$$g_3=\frac{1}{\sqrt{N_1^2+N_2^2}}[N_2,-N_1,0], \tag{A8}$$

$$g_3^{\perp}=\frac{1}{\sqrt{N_1^2+N_2^2}}[N_1,N_2,0], \tag{A9}$$

where $N_m=x_m/r$ is the unit direction vector to the receiver.

[1] S. Crampin and S. C. Kirkwood, ''Shear-wave singularities of wave propagation in anisotropic media,'' J. Geophys. **49**, 43–46 (1981).
[2] T. Mensch and P. Rasolofosaon, ''Elastic-wave velocities in anisotropic media of arbitrary symmetry-generalization of Thomsen's parameters ε, δ and γ,'' Geophys. J. Int. **128**, 43–64 (1997).
[3] J. E. Zimmer and J. R. Cost, ''Determination of the elastic constants of an unidirectional fiber composite using ultrasonic velocity measurements,'' J. Acoust. Soc. Am. **47**, 795–803 (1970).
[4] T. Lhermitte and B. Perrin, ''Anisotropy of elastic properties of cross-ply fiber-reinforced composite materials,'' IEEE Ultrasonic Symposium Proc. **2**, 825–830 (1991).
[5] B. K. Hoffmeister, S. M. Handley, S. A. Wickline, and J. G. Miller, ''Ultrasonic determination of the anisotropy of Young's modulus of fixed tendon and fixed myocardium,'' J. Acoust. Soc. Am. **100**, 3933–3940 (1996).
[6] P. L. Kuo, P. C. Li, and M. L. Li, ''Elastic properties of tendon measured by two different approaches,'' Ultrasound Med. Biol. **27**, 1275–1284 (2001).
[7] H. S. Yoon and J. L. Katz, ''Ultrasonic wave propagation in human cortical bone-I. Theoretical considerations for hexagonal symmetry,'' J. Biomech. **9**, 407–412 (1976).
[8] S. F. Levinson, ''Ultrasound propagation in anisotropic soft tissues: The application of linear elastic theory,'' J. Biomech. **20**, 251–260 (1987).
[9] J. Anderson, ''Elasticité musculaire longitudinale et transversale: influence de l'abscence de desmine—Longitudinal and transversal elasticity of muscles: influence of desmin lack,'' thesis (in French), Compiègne University of Technology, France, 2000.
[10] S. F. Levinson, M. Shinagawa, and T. Sato, ''Sonoelastic determination of human skeletal muscle elasticity,'' J. Biomech. **28**, 1145–1154 (1995).
[11] F. Lee, J. P. Bronson, R. M. Lerner, K. J. Parker, S. R. Huang, and D. J. Roach, ''Sonoelasticity imaging: Results in *in vitro* tissue specimens,'' Radiology **181**, 237 (1991).
[12] S. Catheline, F. Wu, and M. Fink, ''A solution to diffraction biases in sonoelasticity: The acoustic impulse technique,'' J. Acoust. Soc. Am. **105**, 2941–2950 (1999).
[13] L. Sandrin, M. Tanter, J. L. Gennisson, S. Catheline, and M. Fink, ''Shear elasticity probe for soft tissue with 1D transient elastography,'' IEEE Trans. Ultrason. Ferroelectr. Freq. Control **49**, 436–446 (2002).
[14] D. C. Gakenheimer and J. Miklowitz, ''Transient excitation of an half space by a point load traveling on the surface,'' J. Appl. Mech. **36**, 505–514 (1969).

[15] V. Vavryčuk, ''Exact elastodynamic Green's functions for simple types of anisotropy derived from higher-order ray theory,'' Stud. Geophys. Geod. **45**, 67–84 (2001).
[16] L. Sandrin, ''Elastographie impulsionnelle par ultrasons: du palpeur acoustique à l'imagerie ultrarapide—Ultrasound based transient elastography: from the shear elasticity probe to the ultrafast imaging,'' thesis (in French), Paris VI University, France, 2000.
[17] J. Lepetit, ''Deformation of collagenous, elastin and muscle fibres in raw meat in relation to anisotropy and length ratio,'' Meat Sci. **26**, 47–66 (1989).