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Power law decay of zero group velocity Lamb modes

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Abstract

Elastic plates or cylinders can support guided modes with zero group velocity (ZGV) for nonzero wave numbers. At these ZGV-points of the dispersion curves the acoustic energy does not propagate in the waveguide, resulting in sharp resonance effects. In this paper, using laser-based ultrasonic techniques, we investigate the time-decay of the mechanical displacement for ZGV Lamb modes excited by a pulsed laser in various thin metallic plates. In the first microseconds of the local plate vibration, we observed a $t^{-1/2}$ decay due to the second order term in the dispersion relation. This effect is dominant because the first order term, proportional to the group velocity, vanishes for ZGV-modes. After this power law decay, the mechanical displacement undergoes an exponential decay corresponding to the wave damping. Then, the local attenuation of the plate material can be estimated at the ZGV-resonance frequency.

Keywords: Lamb modes; Zero group velocity; Time decay

1. Introduction

Some branches of the dispersion curve $\omega(k)$ of modes propagating in homogeneous waveguides present points where the group velocity vanishes for a nonzero wave number k_0. Such modes exhibit unusual properties such as backward-wave propagation, interferences between backward and forward modes, resonance and ringing effects.

The so-called "backward-wave" propagation, which occurs in the negative-slope region $(k < k_0)$ where group velocity V_g and phase velocity V have opposite signs, has been reported both in elastic and optical waveguides [1,2]. In acoustics, this phenomenon was observed in homogeneous elastic plates having free surfaces [3] or in fluid loaded hollow cylinders [4]. In optics, it was shown recently that such anomalous modes can be created in waveguides with an arbitrary cross section, provided that the outer surface was coated with a reflective cladding [5].

Since at frequencies corresponding to zero group velocity (ZGV) points, the energy does not propagate, sharp resonance peaks and ringing effects was expected early [6] and observed recently [7]. Using laser-based ultrasonic techniques, these ZGV resonances were observed for the S_1-Lamb mode and also for the second

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antisymmetric (A_2) mode propagating in a plate [8]. Recently, we have shown that these ZGV modes, excited by a laser pulse and optically detected in the time domain, can be exploited for measuring locally the mechanical properties of isotropic materials [9,10].

In this paper, we investigate another phenomenon: the power law decay of ZGV Lamb modes due to the second order term in the dispersion relation.

2. Zero group velocity Lamb mode resonances

The propagation of Lamb waves in an elastic plate is represented by a set of dispersion curves giving the angular frequency $\omega = 2\pi f$ of each symmetric (S) and antisymmetric (A) modes versus the wave number $k = 2\pi/\lambda$ [11,12]. In an isotropic solid having a Poisson's ratio ν smaller than 0.45, the first order symmetric (S_1) Lamb mode, exhibits a particular behaviour at frequencies where the group velocity vanishes while the phase velocity remains finite. Fig. 1 shows the dispersion curves of the S_1 and S_2 symmetric Lamb modes for a copper plate of thickness d made of a material having longitudinal and transverse bulk wave velocities equal to $V_L = 4.56$ km/s and $V_T = 2.32$ km/s, respectively. The group velocity $V_g = \frac{d\omega}{dk}$ of the lower S_1-mode vanishes at the frequency-thickness product $f_0 d = 2.085$ MHz mm and for a wavelength $\lambda_0 = 3.86d$. The phase velocity $V = f\lambda$ of this ZGV-mode is equal to $V_0 = 8.05$ km/s and the dispersion curve is well approximated by a parabola in a large range of wave number k about $k_0 = 2\pi/\lambda_0$:

$$\omega(k) \cong \omega_0 + D(k - k_0)^2. \tag{1}$$

The coefficient D (m^2 s^{-1}) is proportional to the transverse wave velocity V_T and to the plate thickness d:

$$D = \delta(\nu)V_T d, \tag{2}$$

where the dimensionless coefficient $\delta(\nu)$ depends only on Poisson's ratio. For copper ($\nu = 0.325$), δ is equal to 0.30. Like in optics [13], the ZGV phenomenon can be explained by a strong repulsion between the two neighbouring S_1 and S_2 symmetric modes near the cut-off frequencies.

In our experiments, Lamb waves are generated by a Q-switched Nd:YAG (Yttrium Aluminium Garnet) laser providing pulses having a 20-ns duration and 4-mJ of energy. The spot diameter of the unfocused beam is equal to 1 mm. The local vibration of the plate is detected at the same point by a heterodyne interferometer equipped with a 100-mW frequency doubled Nd:YAG laser [14]. The calibration factor for mechanical displacement normal to the surface (10 nm/V) was constant over the detection bandwidth (50 kHz–40 MHz). Signals detected by the optical probe were fed into a digital sampling oscilloscope and transferred to a computer.

Since their group velocities vanish, the acoustic energy at the minimum frequencies of Lamb modes is trapped under the source. Then, the spectrum of the mechanical response of a plate to a sudden and localized impact is expected to be dominated by sharp peaks at the ZGV frequencies. As an example, Fig. 2(a) shows the

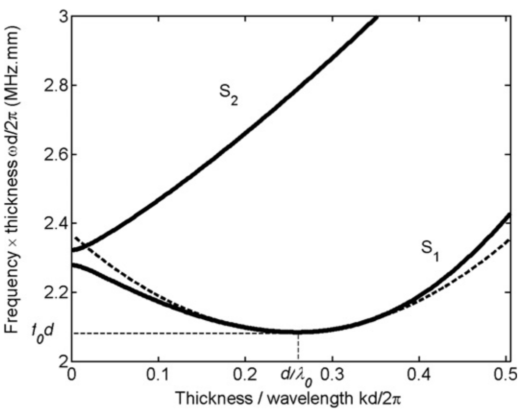

Fig. 1. Dispersion curves of the S_1 and S_2 symmetric Lamb modes propagating in a copper plate of thickness d and bulk wave velocities equal to $V_L = 4.56$ km/s and $V_T = 2.32$ km/s. About the ZGV point, the lower branch is well approximated by a parabola (dashed line).

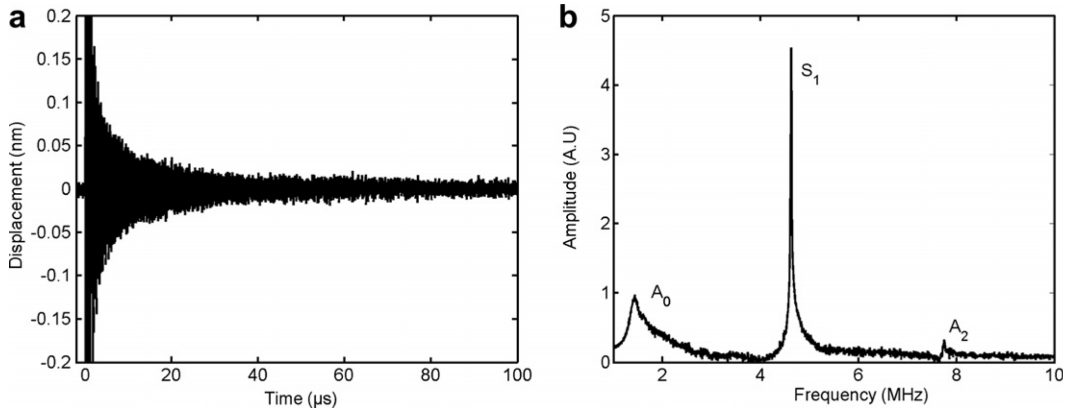

Fig. 2. Mechanical displacement optically generated and detected at the same point on a 0.45-mm thick copper plate. (a) Time signal. (b) Spectrum: the peak at 4.63 MHz corresponds to the minimum of the S_1-Lamb mode dispersion curve.

mechanical displacement optically measured on a 0.45-mm thick copper plate and generated by the laser pulse. The signal oscillates during a long time at a frequency $f_0 = 4.63$ MHz corresponding to the sharp peak observed in the spectrum in Fig. 2(b). This value is very close to the theoretical minimum frequency of the S_1-Lamb mode dispersion curve (Fig. 1):

$$f_0 d = 2.08 \text{ MHz mm and } d = 0.45 \text{ mm} \rightarrow f_0 = 4.62 \text{ MHz}.$$

In the lower frequency range, the spread spectrum of the A_0 mode is cut at frequencies less than 1 MHz by a high-pass filter. The role of this filter is to eliminate the low frequency components due to the heating of the air by the laser energy absorption. The resulting variations of the optical index at the vicinity of the surface create a phase shift along the path of the probe beam, which is detected by the optical probe. At a higher frequency (7.75 MHz), the small peak in Fig. 2b corresponds to the A_2-mode ZGV resonance [9].

In the first microseconds after the laser pulse impact the signal in Fig. 2a exhibits a fast decay, which cannot be explained by the material damping. The mechanical response of a plate to a transient loading has been analysed by several authors [15,16]. They point out large amplitude spikes or resonant responses near the cutoff frequencies of the S_1 and A_2 branches. Since the group velocities of these modes are very small, in far field experiments theirs contributions is observable only on a long time scale. Conversely, in our experiments where the plate vibration is detected in the source area, of the millimetre range, Lamb modes with usual group velocity (2–10 mm/μs) flows out of the source in less than 1 μs. Then, as shown in Fig. 2, the prominent response comes from the ZGV resonant modes, mainly the S_1 one.

3. Analysis and experimental results

In our experiment, the local vibration of the plate is excited in the thermoelastic (linear) regime by the impact of a short laser pulse $q(t)$. In the case of an axisymmetric source having a laser energy distribution $b(r)$, the normal component $u(r,t)$ of the mechanical displacement can be expressed as:

$$u(r,t) = \frac{1}{2\pi} \int_0^{+\infty} C_{\text{th}}(k) Q(\omega) B(k) J_0(kr) e^{i\omega t} k \, dk, \tag{3}$$

where $C_{\text{th}}(k)$ is the thermoelastic conversion coefficient for a given Lamb mode. $Q(\omega)$ is the laser pulse spectrum and $B(k)$ the spatial Fourier transform of $b(r)$.

For $t > 1$ μs, the main contribution to the displacement comes from the modes having a slow group velocity. In the integral (Eq. (3)), the phase $\varphi(k) = \omega(k)t$ undergoes a minimum at the wave number k_0 for which the group velocity $V_g = \frac{d\omega}{dk}$ vanishes. The contribution of each ZGV mode can be calculated by the stationary phase method:

$$u(r,t) = \frac{C_{th}(k_0)}{\sqrt{2\pi\varphi''(k_0)}} Q(\omega_0)B(k_0)J_0(k_0r)k_0 e^{i(\omega_0 t + \pi/4)}, \tag{4}$$

where $\omega_0 = \omega(k_0)$ is the ZGV-resonance frequency. The second derivative of the phase expressed as a function of the local curvature $2D$ of the dispersion curve: $\varphi''(k_0) = 2Dt$.

Setting $A(k_0) = C_{th}(k_0)Q(\omega_0)B(k_0)k_0$, the displacement of the ZGV mode

$$u(r,t) = \frac{A(k_0)}{\sqrt{4\pi Dt}} J_0(k_0r) e^{i(\omega_0 t + \pi/4)} \tag{5}$$

undergoes a $t^{-1/2}$ decay due to the second order term of the dispersion relation. In a dispersive medium, this effect makes the wave packet to spread out as it progresses [17]. The group velocity of Lamb modes also vanishes at the cut-off frequencies. However, since $k_0 = 0$, their amplitude $A(k_0)$ goes to zero.

In order to take into account the material damping, ω_0 is replaced by the complex angular frequency $\omega_0 + ia$. Then, the mechanical displacement undergoes a supplementary exponential decay:

$$u(r,t) = \frac{A(k_0)}{\sqrt{4\pi D}} J_0(k_0r) t^{-1/2} e^{-at} e^{i(\omega_0 t + \pi/4)}. \tag{6}$$

The temporal behaviour of the signal in Fig. 2a is analysed by a short-time Fourier transform at the ZGV-resonance frequency. As shown in Fig. 3, for $t < 10$ μs, the amplitude of the displacement decreases like $t^{-1/2}$, whereas for $t > 30$ μs, an exponential decay corresponding to a viscoelastic mechanism can be observed. With a time-decay constant $\tau = 1/a$ equal to 60 μs, the signal multiplied by $t^{1/2}e^{t/\tau}$ is in average approximately flat.

The same $t^{-1/2}$ power law, responsible of the early decay of the signal, was observed with a 12-mm long narrow line source and for other materials. It proves that diffraction effects can be neglected and that the geometry (1-D or 2-D) of the source does not affect the time-decay law. Fig. 4 shows, in logarithmic scales, the variations of the ZGV-displacements for Duralumin, steel and copper plates having thicknesses in the range 0.45–0.49 mm. For $t < 10$ μs, the average slopes of the curves are close to -0.5, in agreement with the power law decay.

Measuring the time decay of the ZGV-resonance allows for a local estimation of the intrinsic attenuation of the material. Provided that the time-decay constant τ was estimated after the diffusion regime, the attenuation coefficient of the material α can be calculated from the formula

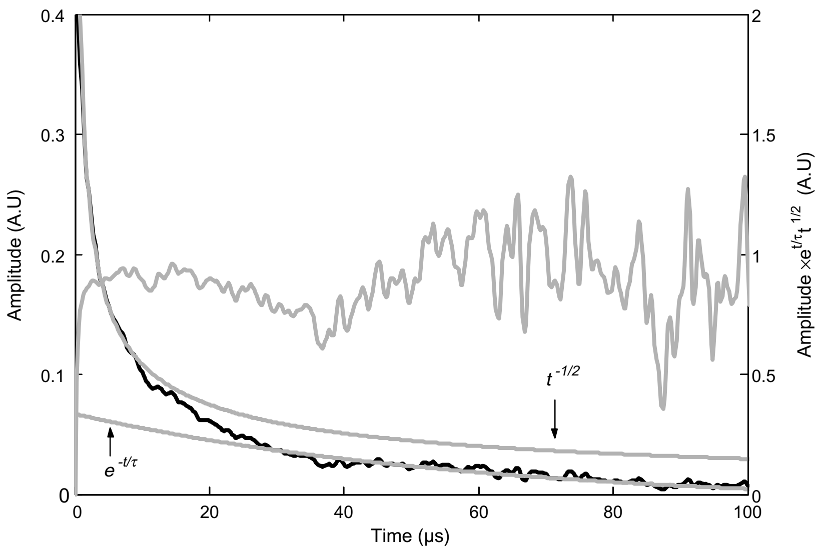

Fig. 3. Time decay of the signal in Fig. 2a analysed by a short-time Fourier transform at the S_1-mode ZGV frequency. The grey curve represents the product of the signal by the function $t^{1/2}e^{t/\tau}$ with $\tau = 60$ μs.

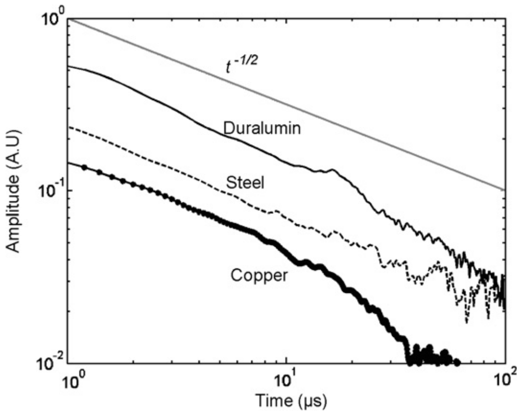

Fig. 4. Experimental time variations, in logarithmic scales, of the ZGV-displacements for Duralumin, steel and copper plates compared to the $t^{-1/2}$ law.

$$\alpha = \frac{1}{V_0 \tau}. \tag{7}$$

V_0 is the phase velocity at the ZGV point, i.e. a combination of bulk wave velocities. Then, α cannot be simply related to longitudinal or shear wave attenuation. For the copper plate, with the measured time-decay constant $\tau = 60$ µs, the value found at the resonance frequency $f_0 = 4.63$ MHz is:

$$\alpha \cong 2.1 \text{ Np/m} = 18 \text{ dB/m}.$$

The same procedure was applied to steel and Duralumin plates. Results are given in Table 1.

In the case of a low loss material such as Duralumin ($\tau = 800$ µs), the major part of the acoustic energy leaks outside the source area during the first regime. Then, the damping process is more difficult to observe. From the time decay of the diffuse ultrasonic field in different aluminum-alloy samples, damping factors as small as 2.2 dB/ms have been measured at 6 MHz by Haberer et al. [18]. Assuming that the diffuse field is dominated by the shear modes [19] propagating at the velocity $V_T = 3.10$ km/s, the value found for the attenuation coefficient (0.7 dB/m) is close to our result (0.9 dB/m). The difference may be ascribed to the scattering from S_1 to other propagating branches at the same frequency.

4. Conclusion

After a local and transient loading of a plate, the acoustic energy decay in the source area can be explained by three mechanisms: the energy transport phenomenon at the Lamb wave group velocity, the material damping and the second order dispersive effect. Generally, the first mechanism dominates the other two. Since at the minimum frequency of the dispersion curve the group velocity vanishes, no energy transport occurs, and the slower other two phenomena can be observed. Using laser-based ultrasonic techniques, we have shown that the mechanical response of thin metallic plates to a laser pulse impact was dominated by the resonance at the zero group velocity point of the S_1-Lamb mode dispersion curve. In the first microseconds, we observed a fast decay of the local vibration amplitude, which cannot be explained by the material damping. We

Table 1
Attenuation coefficient α of metallic plates estimated (with ±20% error) by the ZGV-resonance method

Material	d (mm)	f_0 (MHz)	V_0 (km/s)	τ (µs)	α (dB/m)
Copper	0.45	4.63	8.05	60	18
Steel	0.45	6.14	10.5	400	2.1
Duralumin	0.49	5.86	11.2	800	0.9

d is the plate thickness, f_0 the resonance frequency, V_0 the S_1-ZGV phase velocity and τ the time-decay constant.

demonstrated that this effect is ascribed to the second order term in the dispersion relation. This phenomenon, always present, is highlighted in our experimental conditions because the first order term, i.e. the group velocity, is equal to zero. Furthermore, we show that the local attenuation of the material can be measured without any mechanical contact.

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