Analysis of surface acoustic wave propagation on a cylinder using laser ultrasonics

D. Clorennec[a)] and D. Royer
Laboratoire Ondes et Acoustique, Université Paris 7-CNRS-ESPCI 10, rue Vauquelin, 75231 Paris Cedex 05-France

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This letter shows an unexpected phenomenon for surface acoustic waves (Rayleigh waves) propagating on a solid cylinder. It was observed that a short Rayleigh wave pulse was reversed during its propagation. In experiments on duraluminum and steel cylinders, transient surface acoustic waves were launched by a pulsed YAG laser focused along a line. The radial component of the displacement was measured by a heterodyne optical interferometer. The wave forms recorded versus the angle θ between the source and the detection point shows that the Rayleigh pulse, which is monopolar (positive) near the source, becomes bipolar at $\theta=45°$ and monopolar (negative) at $\theta=90°$. An analytical calculation taking into account the dispersive effect reproduces wave forms whatever the propagation distance. A physical model is proposed to account for the reversal of the acoustic pulse. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1586463]

It is of great importance to test cylindrical parts such as rotating axes of machines. Surface-breaking or subsurface defects can be detected with surface acoustic waves[1] (SAWs). The difficulty to apply conventional piezoelectric transducers on a curved surface[2] can be overcome by the use of lasers, which does not require any mechanical contact with the inspected surface. Moreover, optical generation and detection of elastic waves can be achieved in a large bandwidth.[3] In previous works, laser-based ultrasonic techniques have been used to study the SAW propagation on spheres and cylinders.[4] On spheres, the phase jump of a wave passing through a pole was especially analyzed.[5] Recently, diffraction-free propagation of SAWs on a sphere has been reported.[6]

The propagation of SAWs on a cylinder, of radius a, is different from their propagation on a plane surface. Cylindrical surface waves fall into two categories: the Rayleigh and the so-called whispering gallery waves. The characteristic equation providing the angular frequency ω versus the wave number k is given in Viktorov's book.[7] An integer value n of ka corresponds to a resonance frequency ω_n. The lowest mode, defined for $n\geqslant 1$, corresponds to the Rayleigh mode and the others, defined for $n\geqslant 0$, to the whispering gallery modes. Figure 1 shows, for a duraluminum cylinder, the Rayleigh wave phase velocity V versus the product ka. The dispersive effect is mostly noticeable for low frequencies ($ka<10$) such that the wavelength λ_R is larger than $a/2$. As ka tends to infinity (large frequencies), the velocity tends to the value $V_R=2930$ m/s, corresponding to the speed of the Rayleigh wave in a duraluminum plate. The high temporal resolution of laser ultrasonic technique enables us to study the dispersion effect on the high frequency components.

Surface waves were generated by a Q-switched Nd:YAG laser providing pulses having a 24-ns duration and a 3-mJ energy. A beam expander and a cylindrical lens (focal length $F=200$ mm) were used to focus the beam onto a 6-mm-wide line, parallel to the cylinder axis (Fig. 2). The mechanical displacement normal to the surface was measured by a heterodyne interferometer[8] equipped with a 2-mW He-Ne laser (SH 130 optical probe from Thales Laser). The calibration factor (100 mV/nm) was constant over the detection bandwidth (50 kHz–20 MHz). Signals detected by the optical probe were fed into a digital sampling oscilloscope (Tektronix™ 520D) and transferred to a computer. The probe beam can rotated around the sample by a step of 5°.

Experiments were carried on duraluminum cylinders of diameter 12, 25, and 50 mm and on a steel cylinder of diameter 25 mm. The first arrival of the Rayleigh wave propagating in a clockwise direction was recorded according to the angle between the source and the detection point from $\theta=5°$ to $\theta=180°$. Figure 3 shows a series of wave forms versus the angle θ and time t on a duraluminum cylinder of diameter 25 mm. We observed significant modifications of the Rayleigh wave form. Near the source, the mechanical displacement is a monopolar pulse (positive if the normal points towards the inner of the cylinder), it becomes bipolar at $\theta=45°$ and monopolar (negative) at $\theta=90°$. When the generation and detection zones are diametrically opposite

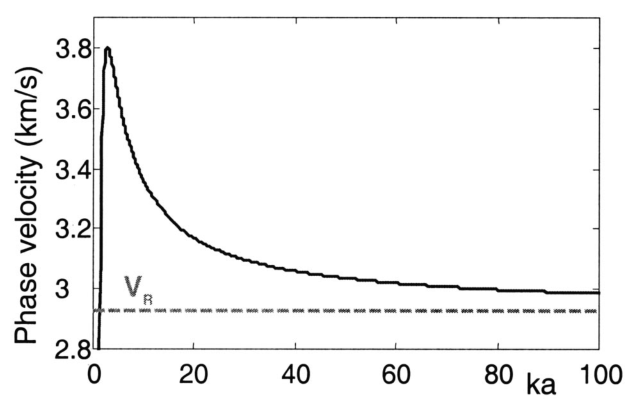

FIG. 1. Duraluminum cylinder. Rayleigh wave phase velocity (V) versus ka.

a)Electronic mail: dominique.clorennec@loa.espci.fr

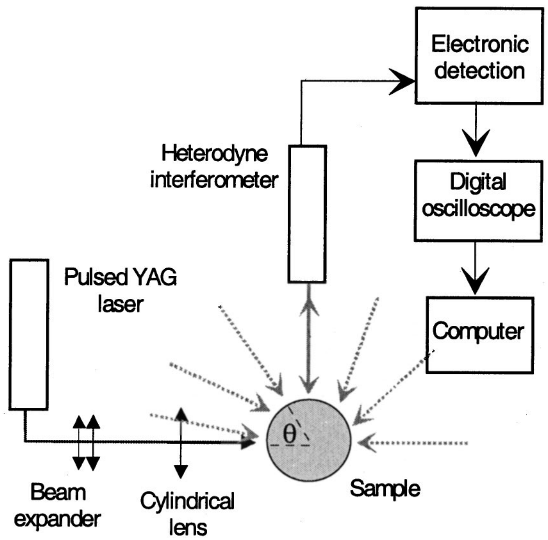

FIG. 2. Experimental setup.

($\theta = 180°$), the pulses for the clockwise and counterclockwise propagations merge into a single peak. The amplitude of the displacement is increased twofold.

In order to explain this evolution, we have developed a one-dimensional analytical calculation taking into account the dispersion effect. In a diametrical plane, the source is considered as a point. At a given angle θ, the mechanical displacement u associated to the Rayleigh wave propagating in a clockwise direction is given by

$$u(\theta,t) = A\int_{-\infty}^{+\infty} Q(\omega)e^{i\omega[t - s(\omega)a\theta]}d\omega. \tag{1}$$

$s(\omega) = 1/V(\omega)$ is the Rayleigh wave slowness, which depends on the angular frequency (Fig. 1). $Q(\omega) = 1/(1 + i\omega\tau)^2$ is the spectrum of a normalized function $q(t)$ representative of the laser pulse shape ($\tau = 10$ ns). Figure 4 shows a good agreement between calculated and experimental signals. The theoretical angle for which the mechanical displacement is reversed is found to be $\theta_r = 90°$. This calculation shows that for short propagation distance, diffraction effects can be neglected.

In order to explain the origin of this displacement reversal, we propose a physical model taking into account the dispersion effect for large ka values. The spectrum of the

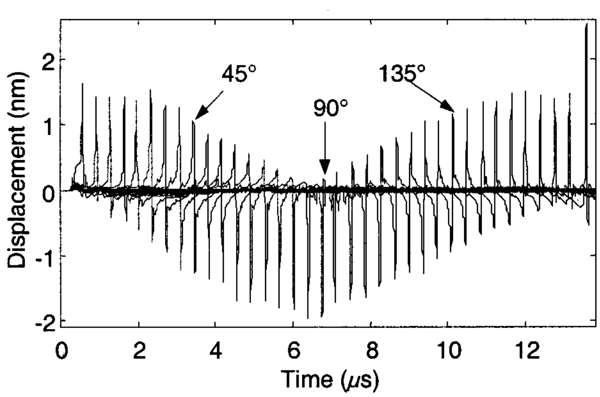

FIG. 3. Rayleigh wave forms measured on a 25-mm-diameter duraluminum cylinder versus the detection angle θ from 5° to 180°.

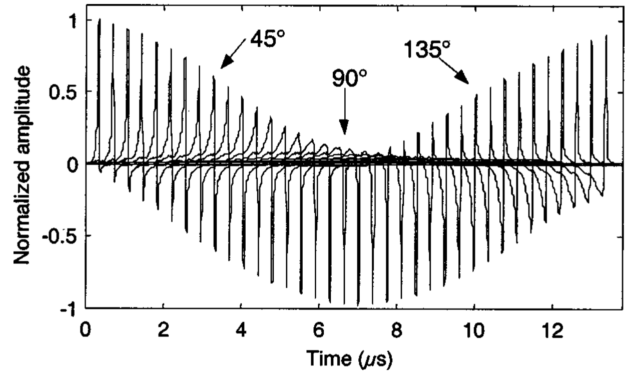

FIG. 4. Rayleigh wave forms calculated for a 25-mm-diameter duraluminum cylinder versus detection angle θ.

laser-generated acoustic pulse spreads from 0.5 to 20 MHz, which corresponds to $15 < ka < 520$. For such high ka values, the asymptotic behavior of the dispersion curve can be approximated by

$$V = V_R\left(1 + \frac{\varepsilon}{ka}\right), \tag{2}$$

where ε is a constant involving the wave numbers $q_1 = k_R\chi_1$ and $q_2 = k_R\chi_2$, with

$$\chi_1 = \left(1 - \frac{V_R^2}{V_L^2}\right)^{1/2} \quad \text{and} \quad \chi_2 = \left(1 - \frac{V_R^2}{V_T^2}\right)^{1/2}, \tag{3}$$

where V_L and V_T are the longitudinal and transverse bulk wave velocities, respectively.

From the formula given in Ref. 9, we established that

$$\varepsilon = \frac{(\chi_1 + \chi_2)(\alpha + \alpha^{-1}) + \dfrac{\chi_1^2 - \chi_2^2}{2\chi_1\chi_2}(\alpha - \alpha^{-1}) - 2\chi_2^2\chi_1\beta}{(\chi_1 - \chi_2)(\alpha - \alpha^{-1}) + 4\chi_1\chi_2\beta}, \tag{4}$$

where

$$\alpha = \left(\frac{V_L}{V_T} \cdot \frac{1 + \chi_1}{1 + \chi_2}\right)^2 \tag{5}$$

and

$$\beta = \frac{V_L^2 - V_T^2}{V_R^2}. \tag{6}$$

For duraluminum ($V_L = 6340$, $V_T = 3140$, and $V_R = 2930$ m/s), it was found that $\varepsilon = 1.99$.

From Eq. (2), the slowness can be approximated by

$$s(\omega) \approx s_R - \frac{\varepsilon}{\omega a}. \tag{7}$$

The phase lag of the Rayleigh wave

$$\varphi = -\omega s a\theta = \varphi_R + \varepsilon\theta, \tag{8}$$

is the sum of the phase lag $\varphi_R = -\omega s_R a\theta$ due to the propagation at the constant velocity V_R and of an additional phase-shift $\varphi_a = \varepsilon\theta$ due to the dispersion in the high-frequency range. For an angle θ_r such that $\varphi_a = \pi$, the mechanical displacement is reversed. This value $\theta_r = \pi/\varepsilon$, equal to 90° for duraluminum, is independent of the cylinder radius a. For $\theta = 45°$ and 135°, this model, based on the dispersive effect

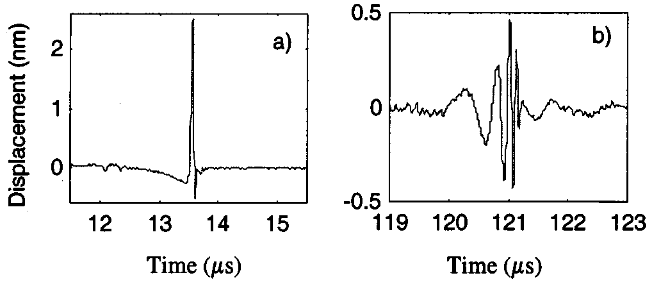

FIG. 5. Rayleigh wave forms detected on a 25-mm-diameter duraluminum cylinder (probe position $\theta = 180°$); (a) first turn and (b) fifth turn.

for large ka, predicts, respectively, phase-shifts of $\pi/2$ and $3\pi/2$, which induce a bipolar Rayleigh wave form.

The same experiment on a duraluminum cylinder of diameter 12 mm shows a phase reversal for $\theta = 90°$ that confirms the independence of θ_r versus the cylinder radius. However, for a cylinder of diameter 50 mm, the phase reversal occurs at $\theta = 70°$. For such a large diameter, the diffraction effects cannot be neglected. According to the diffraction model developed in Ref. 10 for a plane surface, the Rayleigh pulse becomes bipolar in the far field. Assuming, for the line source, a Gaussian energy distribution of width $2b$, this condition is fulfilled for $x_L = a\,\theta_L \geqslant b^2/V_R\Delta$, where Δ is the laser pulse duration. The full length of the line source at $1/e$ of the maximum was found to be $2b = 4$ mm. Using experimental values: $\Delta = 24$ ns, the limit between the near-field and the far-field domains is $x_L = 57$ mm, which corresponds, for a 50-mm-diameter cylinder, to an angle $\theta_L = 130°$ belonging to the observation domain ($5° < \theta < 180°$). In this case, both dispersion and diffraction effects contribute to the bipolarity of the Rayleigh pulse.

The parameter ε defined by Eq. (4) was found to be very insensitive to materials parameters. For example: $\varepsilon = 1.95$ for steel and $\varepsilon = 2.00$ for brass. An experiment performed in similar conditions on a steel cylinder 25 mm in diameter leads to a reversal of the Rayleigh pulse for the same angle $\theta_r = 90°$.

The phenomenon is periodic: the Rayleigh pulse becomes again negative at $\theta = 270°$. After many turns of propagation, the Rayleigh wave form undergoes another modification due to the variation of the phase velocity in the low-frequency range. In an experiment, the probe beam is placed at the opposite of the source ($\theta = 180°$) and the Rayleigh wave form is analyzed according to the number of turns (Fig. 5). The time delay between the two successive pulses corresponding to the first and fifth passages is approximately 107.4 μs, which gives an average velocity of 2930 m/s close to V_R. According to the dispersion curve in Fig. 1, low-frequency components propagate at a higher velocity than high-frequency ones. This spreading of the pulse, which increases with the propagation distance, mixes up with the periodic reversal described earlier.

Owing to its high temporal resolution, laser ultrasonic experiments are an approximate method to study the evolution of surface acoustic waves propagating on a solid cylinder. Important modifications of the Rayleigh wave form versus the detection point have been observed for small angles. Results of a model based on the dispersion effect are in good agreement with the experiments. The reversal of the SAW pulse has been explained by the role of the high-frequency components of the laser-generated acoustic pulse.

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