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Normal-mode theory of nonspecular phenomena for a finite-aperture ultrasonic beam reflected from layered media

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A normal-mode formalism is developed to describe the nonspecular effects of a finite-aperture ultrasonic beam incident onto layered elastic media. Analytical expressions for the reflected field have been obtained for various structures. This model proposed a unique physical picture and resolved the conflict between various explanations made for the nonspecular reflection phenomena. Novel features of leaky wave fields were observed at interfaces of liquid-solid and liquid-solid-liquid structures. These results may be helpful for nondestructive evaluation of layered structures and determination of material signatures in the acoustic microscope. © *1997 American Institute of Physics.* [S0003-6951(97)00702-X]

The nonspecular reflection phenomena of a finite aperture ultrasonic beam incident at a critical angle from a liquid onto layered elastic structures have been the subject of numerous theoretical and experimental investigations over the last four decades.[1–12] Following the discovery of analogous effects in optics by Goos and Hächen, ultrasonic nonspecular reflection effects were originally studied by Schoch.[1,2] When a beam is incident at a critical, phase match to a guided wave supported by layered structure, a portion of the incident energy is transferred into the excited wave. As it propagates along the liquid-solid interfaces, this guided wave leaked back into liquids to form part of the reflected beam. For illustrative purposes, a typical nonspecularly reflected field is schematically shown in Fig. 1, where characteristic regions are delineated. In addition to the lateral (Schoch's) displacement, the resultant reflected field includes a null area and a trailing field that extends along the interface away from the specular (geometrical) reflection region. Experimental demonstrations have indicated the possible utility of the nonspecular reflection phenomena in material characterization and detection of surface or subsurface defects.[7,8] In recent years, the leaky surface waves have received considerable attention because of its usefulness in advanced nondestructive testing techniques such as acoustic microscopy. It has been established that leaky surface waves play a critical role in the determination of material signatures in the acoustic microscope.[13–16]

An important step in the understanding of the nonspecular phenomena was made by Bertoni and Tamir.[3] They developed an analytical model to describe the resultant reflected acoustic field using a superposition of incident *plane* waves reflected from the liquid-solid (L-S) interface. Owing to a suitably simplified reflection coefficient, this approach have an accurate expression for the reflected field composed of two components. They indicated that the nonspecular reflection profiles is caused by the interference of a specularly reflected beam and a leaky surface wave. Their theory provides a physical explanation of the observed ultrasonic intensity null point, a π phase reversal of the field on either side of the null, followed by a trailing ultrasonic field.[4,5,7–9] This method of analysis was employed later by other researchers to extend investigations to various layered elastic structures.[6,8,10–12]

Although the explanation given by Bertoni and Tamir for the results of the L-S reflection has generally been considered as the physical cause of the nonspecular effects and confirmed by experiments on various layered structures, their formalism, unfortunately, could not lead to the same physical picture as in the case of multilayers such as a liquid-solid-liquid (L-S-L) structure. In contrast to the Rayleigh-angle incidence, as shown by Pitts *et al.*,[6,10] the reflection coefficient of a *plane* wave when incident at a plate mode (Lamb) angle is zero due to transmission. As a result, the component devoted to a specular reflection in the previous formalism is absent and the Lamb-type reflections can no longer be described as the interference of a leaky guided mode excited by the incident beam with a specularly reflected beam! Consequently, these authors proposed to describe the Rayleigh- or Lamb-type reflections as interference of the reflected infinite plane waves which make up the incident beam. It should be recalled that both types of nonspecular reflections have similar reflection profiles, and both types of nonspecular reflections are caused by the same reflection coefficient. Therefore, it is preferable to have a model that resolves the conflict between various physical explanation for the nonspecular phenomena. Moreover, a clear understanding of the nonspecular phenomena, especially features of leaky surface waves, can be very helpful in developing efficient techniques for measuring and monitoring properties of layered elastic

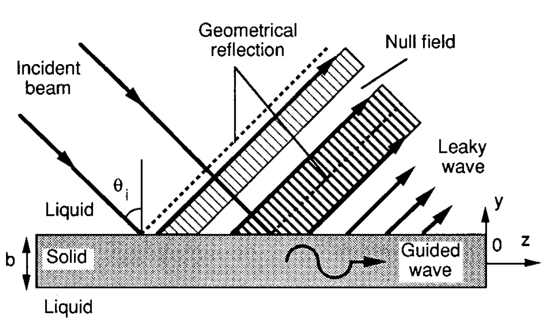

FIG. 1. Nonspecularly reflected field of a bounded beam incident at a critical angle corresponding to a guided mode of the layered structure.

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structures used in many modern and high performance systems.

In this letter, we develop a simple normal-mode formalism to describe the nonspecular phenomena for various structures of planar layered media. Unlike previous theories, this model does not require calculating the reflection coefficients that become extremely difficult, if not impossible, for multilayered solids.[11] Using a perturbation method, this model provides unified analytical expression of the reflected field in terms of properties of normal modes supported by layered elastic structures. A clear and unique interpretation of the physical mechanism is obtained. Several novel features of leaky wave fields will also be discussed.

Let us first examine the leaky wave excitation in a layered elastic medium by the incident beam. For the sake of simplicity, we consider a 2-dimensional problem in that the excited mode propagates in the yz plane along the z direction and is uniform along the y direction, i.e., $\partial/\partial x=0$. The wave amplitude $a_n(z)$ satisfies the normal mode equation,[17,18]

$$\left(\frac{d}{dz}+ik_n\right)a_n(z)=f_n(z)/4P_n, \tag{1}$$

where k_n represents the wave number of the nth propagating mode, P_n is the associated average power flow, and f_n is the loading applied on the boundaries of the elastic waveguide. In the case where the elastic medium is immersed in a nonviscous liquid, only normal stress T_{yy} is transferred onto the upper and lower surfaces of the waveguide. Assume that the width of the beam is much greater than the wavelength in the liquid $(2w \gg \lambda)$ and strikes the waveguide at an angle defined by Snell's law $\theta_n=\arcsin(k_n/k_0)$, where k_0 is the wave number in the liquid, only the nth normal mode is cumulatively excited.[19] If the incident, reflected, and transmitted waves have particle displacements u_I, u_R, and u_T, respectively, the boundary conditions to be satisfied at $y=b/2$ are

$$(u_I+u_R)\cos\theta_n=a_n u_{ny}(b/2), \tag{2a}$$

$$T_{yy}(b/2,z)=-i\omega Z_0(u_I+u_R), \tag{2b}$$

and at $y=-b/2$ are

$$u_T\cos\theta_n=a_n u_{ny}(-b/2), \tag{3a}$$

$$T_{yy}(-b/2,z)=-i\omega Z_0 u_T, \tag{3b}$$

where Z_0 is the acoustic impedance of the liquid. Writing Eqs. (2a) and (3a) implies that the displacement field of the leaky mode in the presence of the liquid loading is approximated as that of a normal mode obtained in the stress-free condition. This perturbation analysis is valid in most practical cases where a liquid density is much smaller than those of layered solid media. By eliminating the reflected and transmitted components u_R and u_T from Eqs. (2) and (3), one obtains the following mode equation governing $a_n(z)$:[18,20]

$$\left[\frac{d}{dz}+ik_n+\alpha_n\right]a_n(z)=-\frac{d\omega^2 Z_0 u_{ny}^*(b/2)u_I(0,z)}{2P_n}, \tag{4}$$

where the parameter $\alpha_n=\alpha_{n1}+\alpha_{n2}$ corresponds to the leak rate or attenuation per unit length when the liquid is present at the upper $(y=b/2)$ or both $(y=\pm b/2)$ L-S interfaces,

$$\alpha_{n1,n2}=\omega d Z_0|u_{ny}(b/2)|^2/(4P_n\cos\theta_n). \tag{5}$$

Here α_{n1} and α_{n2} represent the leak rate of a guided wave through the upper and lower L-S interfaces. In the case of a symmetric waveguide, such as a uniform elastic plate, either symmetric or antisymmetric modes are possible with respect to the meridian plane $(y=0)$. As a result, the leak rate through the upper and lower boundaries shall be the same $\alpha_{n1}=\alpha_{n2}=\alpha_n/2$. In order to compare our results with previous theories, we consider the incidence of a Gaussian beam with effective aperture of $2w$. The length of its projection along the z direction is $w_0=w/\cos\theta_n$,

$$u_I(0,z)=U_I\exp[-(z/w_0)^2-ik_n z]. \tag{6}$$

By defining $a_n=A_n\exp(-ik_n z)$ and substituting $u_I(0,z)$ in Eq. (4), the amplitude of the excited leaky mode $A_n(z)$ in the presence of the liquid can be determined from[21]

$$A_n(z)=-u_I(0,z)(\sqrt{\pi}/4P_n)\omega^2 w_0 Z_0 u_{ny}^*(b/2) \times\exp(\gamma_n^2)\mathrm{erfc}(\gamma_n) \tag{7}$$

with $\gamma_n=-z/w_0+\alpha_n w_0$.

Now it is possible to examine the contribution of the leaky wave to the total reflected field. From Eq. (2a), the reflected field can be readily expressed as

$$u_R(0,z)=u_{\mathrm{SP}}(0,z)+u_{\mathrm{LW}}(0,z), \tag{8}$$

as a sum of two components in that $u_{\mathrm{SP}}(0,z)=-u_I(0,z)$ obviously corresponds to a specularly reflected field. Using Eqs. (2a) and (5), the second one u_{LW} is given by the relation

$$u_{\mathrm{LW}}(0,z)=\sqrt{\pi}(\alpha_{n1}w_0)u_I(0,z)\exp(\gamma_n^2)\mathrm{erfc}(\gamma_n)], \tag{9}$$

which should represent the radiation of the leaky wave. In the case of a L-S structure, one has $\alpha_{n2}=0$ and $\alpha_{n1}=\alpha_n$ equal to the attenuation of the leaky surface wave. The complete reflected field determined by Eqs. (8) and (9) thus reduces to the identical result given by Bertoni and Tamir for

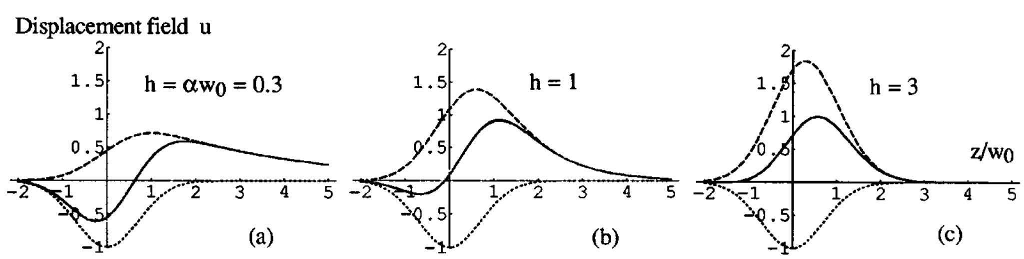

FIG. 2. Resulting reflected fields of displacement (solid curve) along the upper liquid/solid interface of a L/S structure for the case $h=0.3$ (a), $h=1$ (b), and $h=3$ (c). The nonspecular effects are caused by the interference of a specularly reflected field u_{SP} (short-dashed) and a leaky wave field u_{LW} (long-dashed).

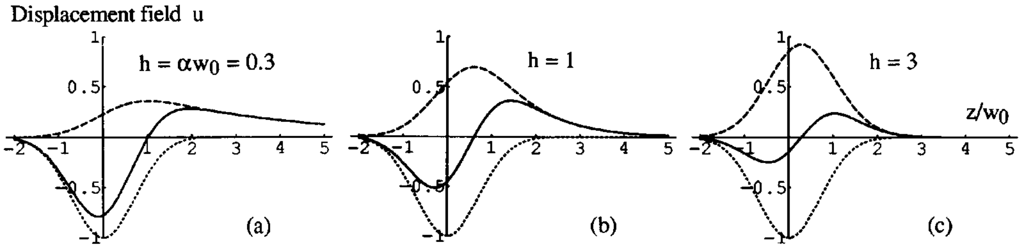

FIG. 3. Similar reflected fields of displacement to Fig. 2, along the upper liquid/solid interface of a L/S/L structure for the case $h=0.3$ (a), $h=1$ (b), and $h=3$ (c).

the Rayleigh-type reflection from a L-S interface.[3] When a bounded beam is incident at a critical angle onto a layered elastic structure such as a solid plate immersed in water (L-S-L structure), as shown above, $\alpha_{n1}=\alpha_n/2$ is half of the leak rate associated with a Lamb mode radiating through the upper and lower surfaces. The corresponding reflected field deduced from Eqs. (8) and (9) is the same as the previous result given in the specific case of a L-S-L structure,[6] except that the result obtained in this analysis is more general and '*S*' denotes any symmetrical layered structure.

In addition to the extension developed for various layered elastic structures, this analytical model provides a unique interpretation of the physical mechanism of the nonspecular reflection effects. As shown in Eqs. (8) and (9), for various layered elastic structures, the resultant reflected field can still be described as interference of a specularly reflected field and a leaky guided wave. However, it is noted that the leaky wave field u_{LW} obtained in this analysis [Eq. (9)] is different from that given in the previous theory.[3] Figures 2 and 3 show nonspecular reflection profiles of a Gaussian beam incident onto L-S and L-S-L structures, respectively, with separate contributions of specularly reflected field and leaky wave field. These profiles were calculated for two values of the parameter $h=\alpha_n w_0$, combining the width of the incident beam w, the critical angle defining $w_0=w/\cos\theta_n$ and the leak rate α_n. For a given value of $h\leqslant 1$, the resultant reflected field is generally distorted, including a lateral beam displacement, a null region, and a trailing field, as observed in experiments (Fig. 1). Like the previous theory, a π phase reversal of the field on either side of the null is observed. When increasing $h>1$, the reflected beam becomes a little distorted for the case of a L-S structure and undergoes only a Schoch displacement [Fig. 2(c)]. It is found that in general the strength of leaky waves and resultant reflected fields for L-S structure is much more than those for L-S-L structure due to transmission. The nonspecular effects of transmitted fields will be described elsewhere.[21]

Another important aspect of the theoretical formalism presented in this work is the prediction of several interesting features of leaky wave fields. As shown in Figs. 2 and 3, the leaky wave field extends well beyond the specular region and its maximum shifts to the right of the reflection axis. Unlike the previous prediction,[3] the leaky wave field and the specularly reflected field are shown to be out of phase with each other over all regions of the L-S interface. This result revives the earlier hypothesis of Neubauer.[4] Furthermore, the characteristic features of the leaky wave fields illustrated here are consistent with the qualitative description given by a ray theory for predicting the acoustic material signature of a specimen.[14] The good physical feel offered by this model will enable insight into the parameters affecting measurements in the acoustic microscope.

In summary, a normal-mode formalism was presented for analyzing the nonspecular reflection effects of a Gaussian beam incident at interfaces of layered elastic structures. Owing to a simple perturbation method, the reflected field has been obtained in a unified formulation, composed of a specularly reflected field and a leaky wave field. This model clarifies the inconsistencies that were found in previous theories for the interpretation of the physical mechanism of the nonspecular effects. Complete reflected fields were observed for L-S and L-S-L structures. Interesting features of leaky wave fields have been revealed by the present model, which are consistent with experimental observations. These results may be useful for measurements in the acoustic microscope.

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[1] A. Schoch, Acustica **2**, 18 (1952).
[2] L. M. Brekhovskikh, *Waves in Layered Media*, 2nd ed. (Academic, London, 1980).
[3] H. L. Bertoni and T. Tamir, Appl. Phys. Lett. **2**, 157 (1973).
[4] W. G. Neubauer, J. Appl. Phys. **44**, 48 (1973).
[5] M. A. Breazeale, L. Adler, and G. W. Scott, J. Appl. Phys. **48**, 530 (1977).
[6] L. E. Pitts, T. J. Plona, and W. G. Mayer, IEEE Trans. Sonics Ultrason. **24**, 101 (1977).
[7] G. W. Scott and L. Adler, Mater. Eval. **33**, 54 (1977).
[8] A. H. Nayfeh, D. E. Chimenti, L. Adler, and R. L. Crane, J. Appl. Phys. **52**, 4985 (). **52**, 4985 (1981).
[9] K. W. Ng, T. D. K. Ngoc, I. Molinero, and M. de Billy, Ultrasonics **26**, 44 (1988).
[10] M. Rousseau and Ph. Gatignol, J. Acoust. Soc. Am. **80**, 325 (1986).
[11] T. Kundu, J. Acoust. Soc. Am. **83**, 18 (1988).
[12] D. E. Chimenti, J. G. Zhang, S. Zeroug, and L. B. Felsen, J. Acoust. Soc. Am. **95**, 45 (1994).
[13] R. D. Weglein, Appl. Phys. Lett. **34**, 179 (1979).
[14] W. Parmon and H. L. Bertoni, Electron. Lett. **15**, 684 (1979).
[15] C. F. Quate, in *Scanned Imaging and Microscopy*, edited by E. A. Ash (Academic, London, 1980), p. 23.
[16] T. Kundu, A. K. Mal, and R. D. Weglein, J. Acoust. Soc. Am. **77**, 353 (1985).
[17] B. A. Auld, *Acoustic Fields and Waves in Solids* (Wiley, New York, 1973), Vol. 2.
[18] G. S. Kino, *Acoustic Waves: Devices, Imaging, and Analog Signal Processing* (Prentice-Hall, New Jersey, 1987).
[19] I. A. Victorov, *Rayleigh and Lamb Waves* (Plenum, New York, 1967).
[20] X. Jia, J. Acoust. Soc. Am. (in press).
[21] X. Jia (unpublished).