Separation of interfering acoustic scattered signals using the invariants of the time-reversal operator. Application to Lamb waves characterization

Claire Prada and Mathias Fink
Laboratoire Ondes et Acoustique, Université Paris 7, C.N.R.S. UMR 7587, E.S.P.C.I., 10 rue Vauquelin, 75352, Paris Cedex 05, France

(Received 18 August 1997; accepted for publication 23 April 1998)

The D.O.R.T. method (in French, Décomposition de l'Opérateur de Retournement Temporel) is a scattering analysis technique using arrays of transducers. The method was shown to be effective in detecting and focusing on pointlike scatterers in Prada *et al.* [J. Acoust Soc. Am. **99**, 2067–2076 (1996)]. Here the D.O.R.T. method is extended to other geometries, applying it to an air-filled cylindrical shell embedded in water. It is shown that the diagonalization of the time-reversal operator permits the various elastic components of the scattered field to be extracted. For the considered cylinder, these components are mainly three circumferential waves ($A0$, $A1$, and $S0$ Lamb modes). Each Lamb mode is shown to correspond to an invariant of the time-reversal operator. The dispersion curves of these waves are calculated from the invariants. In particular, the cutoff frequency of the $A1$ mode is found and provides the thickness of the shell. Finally, resonance frequencies of the shell are deduced from the frequency dependence of the eigenvalues of the time-reversal operator. [S0001-4966(98)03408-0]

PACS numbers: 43.30.Jx, 43.40.Rj, 43.60.Pt [DLB]

INTRODUCTION

Detection and identification of a scattering object by remote sensing techniques is an important problem in a variety of applications. Among them, the reconstruction of internal flaws in ultrasonic nondestructive testing and target classification in underwater acoustics have been widely studied. The scattering of acoustic waves by an elastic body that has dimensions greater than a wavelength is a complex phenomenon. The incident wave is converted into several modes that propagate in and around the object. These waves radiate into the surrounding medium and contribute to the scattered field. In order to understand the scattering process and relate it to the characteristics of the scatterer, it is necessary to identify and separate the various contributions to the signal. In some cases, the modes propagate at different velocities so that their contribution to the signals occur at different times, and they can then be distinguished with time-domain techniques using highly resolved broadband pulses. However, signals from various modes may overlap in time, rendering their identification difficult. This is the case for the problem that we examine in this paper, the scattering of ultrasound from an air-filled cylindrical shell immersed in water. The goal of this paper is to show how the construction of the invariants of the time-reversal process permits the separation and identification of the modes which contribute to the scattered field. The construction of these invariants is an essential part of the D.O.R.T. method (in French, Décomposition de l'Opérateur de Retournement Temporel).

Methods for identifying and separating the various modes that contribute to the signal backscattered from a cylindrical shell have been the subject of many theoretical works.[1,2] Experimental methods such as ''The Method of Isolation and Identification of Resonance''[3] and the short pulse methods[4,5] have proven to be useful. In these methods, the phase velocity of the wave is deduced from the vibrational modes of the shell at its resonance frequencies. Optical remote sensing techniques have also been used to generate and detect circumferential waves. These techniques have the great advantage of providing a local measurement of the field.[6] More recently, this problem was studied with time-reversal mirrors.[7,8]

Acoustic time-reversal mirrors have been widely used since they were first described in 1989.[9–15] A time-reversal mirror consists of an array of transmit/receive transducers. Each element of the array is connected to its own electronic circuitry: a receiving amplifier, an A/D converter, storage memory, and a programmable transmitter. All the channels are processed in parallel for the transmission and reception of the ultrasonic waves. Such a system is able to transmit an ultrasonic wave, to detect the backscattered wave, and then to synthesize a time-reversed version of this wave. This process is performed in less than 1 ms, which can be considered as real time for most scattering media. The time-reversal mirror converts a divergent wave into a convergent one. One consequence is that it can be used to focus on a reflective target through an inhomogeneous medium. If the medium contains several scatterers, the process can be iterated in order to focus on the most reflective one.[9,16]

As shown by Thomas,[7] this system can also be used to study scattering by an elastic shell. He has investigated the two-dimensional (2-D) problem of a thin, air-filled cylindrical shell with dimensions $a=10$ mm, $b/a=0.95$, and $ka\approx125$ under normal incidence (a is the outer radii, b the inner radii, and k the wave number in the loading medium). Two waves propagating around the shell were detected: the first can be considered as a S_0 Lamb wave and the second as an A_0 Lamb wave. As the contributions of these circumferential waves occur at different times, they could be selected temporally with a proper time window and then time reversed. This process allowed each wave to be generated separately. The points at which the incident wave converts into circumferential waves were determined and the phase velocity of each wave was deduced from the distance between these points. This method is limited to the case in which circumferential waves do not interfere and thus can be time resolved. In the case considered by Thomas, the frequency range included the cutoff frequency of the second antisymmetrical Lamb wave A_1. This highly dispersive wave interfers with A_0 and S_0 and could not easily be distinguished in the scattered signals. As we shall see now, the D.O.R.T. method solves this problem and allows one to treat all three modes simultaneously.

The D.O.R.T. method was derived from the theoretical study of iterative time-reversal mirrors,[16] and applies to detection and focusing with large arrays of transducers. This method shares some of the principles used in eigenvector decomposition techniques for passive source detection.[17,18] However, these last techniques assume statistically uncorrelated sources and requires averaging of the measured data, while the D.O.R.T. is active and deterministic.

The D.O.R.T. method consists of determining the invariants of the time reversal process. The calculation is performed offline after the measurement of the response functions of the array in the presence of the scattering medium. It is not a real-time procedure; however, it can still be applied in many experimental situations. Its practical advantage is that it does not require the complex programmable emitters of the time-reversal mirrors. As already mentioned, the main interest is that it allows one to separate scattered waves that overlap in time and cannot be separated by time windowing. This method was applied to selective focusing in media containing several targets.[19,20] It was shown that for N pointlike and well-resolved targets of different reflectivities, the number of independent invariants is equal to N. Furthermore, each invariant provides a phase and amplitude law to be applied to the transducer array in order to focus selectively on one of the targets.

In the first part, we review the principles of the D.O.R.T. method. In the second part, the D.O.R.T. method is applied to the detection of Lamb waves on a thin cylindrical shell. It is shown that although the signals due to the various Lamb waves overlap in time, they can nonetheless be separated into the contribution from individual modes. The dispersion curves and the resonance spectrum of the various modes are determined experimentally.

I. PRINCIPLE OF THE D.O.R.T. METHOD

The D.O.R.T. method relies on a matrix formalism describing the scattering experiment. For a given transducer array and a given scattering medium, the time-reversal process is characterized in the frequency domain by a complex matrix. This matrix is obtained from the interelement impulse response functions which can be measured straightforwardly. As shown further, this matrix is Hermitian and its eigenvectors are independent invariants of the time-reversal process. These invariants are related to the structure of the scattering medium and they carry valuable information about this medium. The D.O.R.T. method consists of the determination and analysis of these eigenvectors.

A. The transfer matrix and the time-reversal operator

An array of N transducers placed in front of a time invariant scattering medium is considered as a linear and invariant system of N inputs and N outputs. The N received signals $r_l(t)$, $1 \leqslant l \leqslant N$ depend on the N transmitted signals $e_m(t)$, $1 \leqslant m \leqslant N$ through the linear relation

$$r_l(t) = \sum_{m=1}^{N} k_{lm}(t) \underset{t}{\otimes} e_m(t), \quad 1 \leqslant l \leqslant N. \tag{1}$$

Here, $k_{lm}(t)$ is the impulse response function from the element m to the element l. These interelement response functions take into account all the propagative effects through the medium under investigation as well as the acousto-electric responses of the two elements. In the Fourier domain, Eq. (1) simplifies using a matrix formula:

$$R(\omega) = \mathbf{K}(\omega) E(\omega), \tag{2}$$

where $E(\omega)$ and $R(\omega)$ are the vectors of the Fourier transforms of the transmitted and received signals. $\mathbf{K}(\omega)$ is the $N \times N$ transfer matrix of the system.

The matrix relation between transmitted and received signals leads to an expression for the transmitted signal after two successive time-reversal operations. Let E^0 be the initial input vector signal. The output signal is then $R^0 = \mathbf{K}E^0$.

The time-reversal operation is equivalent to phase conjugation in the frequency domain, so that the new transmitted signal E^1 is the phase conjugate of the previous received signal R^0

$$E^1 = \mathbf{K}^* E^{0*}.$$

The new received signal is then

$$R^1 = \mathbf{K}E^1 = (\mathbf{K}^*\mathbf{K}E^0)^*.$$

$\mathbf{K}^*\mathbf{K}$ is called the time-reversal operator.

As a consequence of the well-known reciprocity theorem the response from element number m to element number l is equal to the response from element number l to element number m, so that the matrix $\mathbf{K}$ is symmetrical. The symmetry of $\mathbf{K}$ implies that $\mathbf{K}^*\mathbf{K}$ is Hermitian positive. In consequence, it can be diagonalized in an orthogonal basis and has real positive eigenvalues. An eigenvector corresponds to an invariant of the time-reversal operation. Two invariants correspond to contributions to the scattered field that are independent solutions of the wave equation. Each of these waves can exist alone even though they are usually superimposed in the experiments.

B. The D.O.R.T. method

The first step of the D.O.R.T. procedure is the measurement of the interelement impulse responses of the system. This measurement can be done with any multiplexed system by $N \times N$ transmit–receive operations. The components of the transfer matrix $\mathbf{K}$ are obtained by a Fourier transform of each signal. The second step is the diagonalization of the

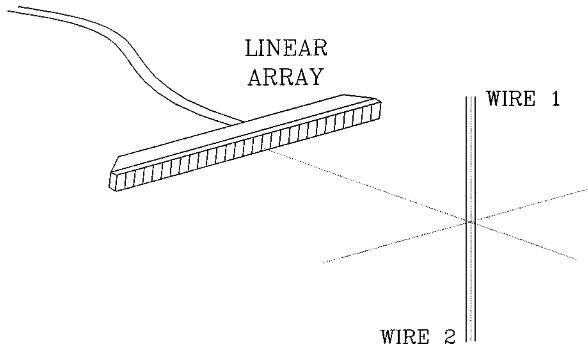

FIG. 1. Experimental setup for two symmetrical wires.

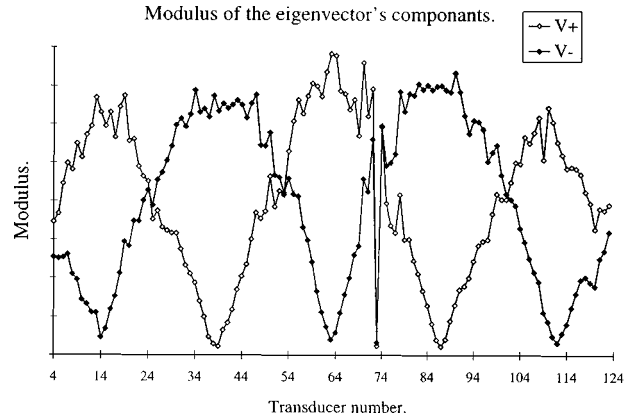

FIG. 2. Modulus of the components of the two eigenvectors versus array element (transducer 73 failed).

time-reversal operator $\mathbf{K}^*\mathbf{K}$ at a chosen frequency. The eigenvalues distribution contains interesting information on the scattering medium. In the case of pointlike scatterers, the number of significant eigenvalues is exactly the number of scatterers provided they are resolved by the system. More generally, this number corresponds to the number of independent secondary sources in the medium.[16,19,20] The third step is to backpropagate each eigenvector. This can be done either numerically or experimentally. In our case, the propagating medium is homogeneous and the numerical backpropagation can be computed.

C. Experiment with two scatterers in a symmetrical geometry

The results presented in Sec. II can be illuminated by the results obtained in the simple case of two identical pointlike scatterers in a symmetrical geometry. The calculation of the eigenvectors for two pointlike scatterers was presented in a recent paper.[9] In the case of two identical scatterers placed symmetrically with respect to the array of transducers, the expression of the eigenvectors simplifies. Let $H_{il}(\omega)$ be the response of scatterer number i ($i=1,2$) to transducer number l ($1 \leqslant l \leqslant N$). We have shown that the two eigenvectors are

$$V_+ = (H_{11}^* + H_{21}^*, \ldots, H_{1N}^* + H_{2N}^*)$$

and

$$V_- = (H_{11}^* - H_{21}^*, \ldots, H_{1N}^* - H_{2N}^*).$$

They correspond to the phase conjugate of the sum and difference of the responses of each scatterer to the array.

In the experiment, a linear array of 128 transducers spaced 0.417 mm and of central frequency 3 MHz was used. The target was made of two wires placed perpendicular to the array in a symmetrical geometry at the depth of 92 mm and separated by 2 mm (Fig. 1). The time-reversal operator is measured for this configuration and its diagonalization at 3 MHz reveals two main eigenvectors V_+ and V_-. The modulus of the components of those two vectors (Fig. 2) correspond to the interference pattern of two sources oscillating in phase (V_+) and opposite phase (V_-). The numerical backpropagation of V_+ and V_- at the depth of the wires confirms this interpretation [Fig. 3(a) and (b)]: the field is focused at the position of the wires, where the phase of the field is equal for V_+ and opposite V_-.

II. APPLICATION TO LAMB WAVES

A. Circumferential waves and time reversal

A circumferential wave propagating in a thin hollow cylinder ($b/a = 0.95$) can be considered a Lamb wave. It is generated at a given angle of incidence θ with respect to the normal to the surface. According to Snell's law, this angle satisfies the relation

$$\sin(\theta) = \frac{c_0}{c_\phi},$$

where c_0 is the sound velocity in water and c_ϕ is the phase velocity of the Lamb wave. While propagating around the cylinder, it radiates into the fluid at the opposite angle with

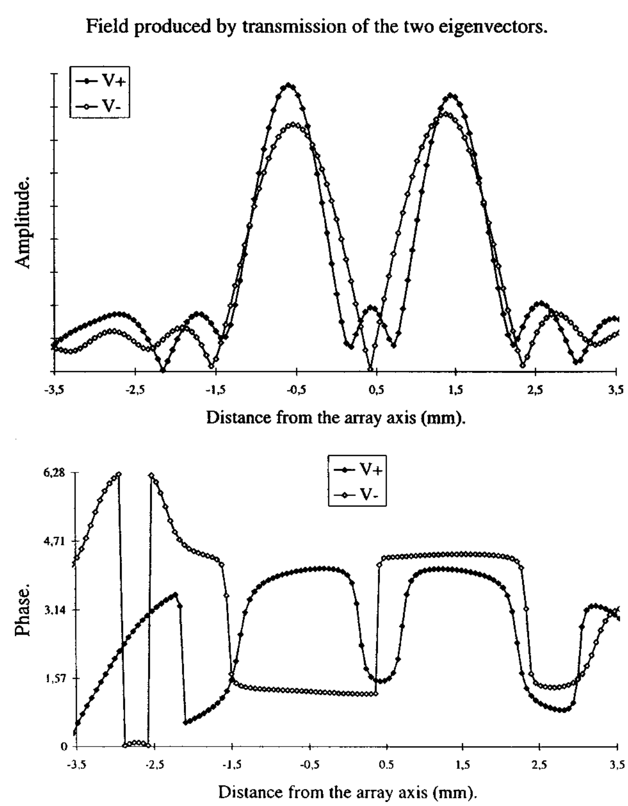

FIG. 3. Amplitude and phase of the field produced by transmission of the two eigenvectors.

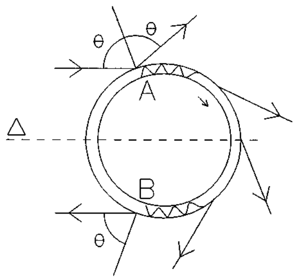

FIG. 4. Generation and radiation of a Lamb wave on a thin hollow cylinder.

respect to the normal to the surface. For a plane wave of incident direction Δ, two Lamb waves are generated at points A and B symmetrical with respect to Δ (Fig. 4). These waves radiate from B and A back to the source. From the direction of observation Δ, the radiated wave appears to be emitted from a pair of secondary sources in A and B. The distance d_{AB} between generation and radiation points A and B is given by the simple relation

$$d_{AB}=D\,\frac{c_0}{c_\phi},$$

where D is the diameter of the cylinder.

Using an array of transducers, a wave can be focused at the surface of the cylinder (Fig. 5). As the wave is locally plane within the focal zone the same approach as before can be used: A Lamb wave is generated at point A if the Snell's law is satisfied. Turning around the shell, the Lamb wave radiates from the opposite point B toward the array. After one time reversal, this wave radiates towards the array from point A. Thus it appears that this wave is invariant under two successive time-reversal processes. Consequently, it should be associated to an eigenvector of the time-reversal operator $\mathbf{K}^*\mathbf{K}$. In fact, due to the symmetry of the system, the focusing in A and B are both associated to the two same eigenvectors just like in the case of two symmetrical pointlike scatterers (Sec. I D).

B. Experiment

The array is linear and made of 96 rectangular transducers. The array pitch is 0.417 mm and the central frequency is

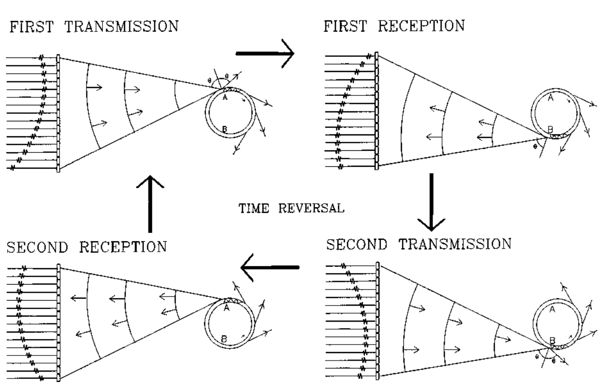

FIG. 5. Lamb waves are invariants of the time-reversal process: A wave focused on point A generates a Lamb wave which radiates towards the array from point B. After two successive time-reversal processes of this Lamb wave, the transmitted wave is similar to the first one, consequently this wave is associated to an invariant of the time-reversal process.

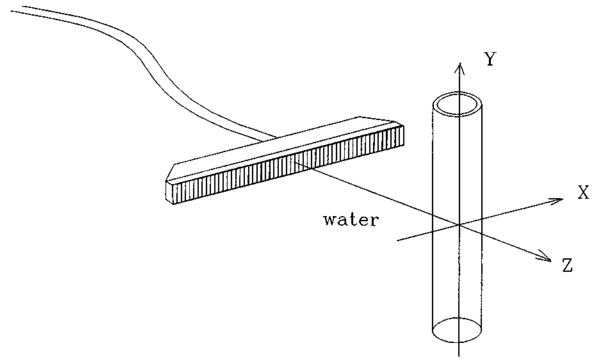

FIG. 6. Experimental setup.

3 MHz with 60% bandwidth. The sampling frequency of received signals is 20 MHz. The steel cylinder of diameter 20 mm and thickness approximately 0.6 mm is placed perpendicular to the array of transducer at a depth of 80 mm symmetrically with respect to the array axis (Fig. 6). For such cylinder and frequency range, the radiation of three Lamb waves should be observed: A_0, S_0, and A_1 as shown on the dispersion curves (Fig. 7).

1. Analysis of the echo

A short pulse is launched by the center element of the array. The echo of the cylinder is recorded on the 96 elements (Fig. 8). The first wavefront corresponds to the strong specular echo. The signal observed later is the elastic part of the echo. Between 15 and 25 μs, two pairs of wavefronts with interference fringes can be distinguished. Those wavefronts correspond to the radiation of two circumferential waves that have turned once around the shell. The one coming first is identified as the S_0 Lamb mode and the second as the A_0 Lamb mode. Interfering with those well-defined wavefronts is the contribution of the highly dispersive A_1 wave.

2. Separation of the modes

To separate these contributions we now apply the D.O.R.T. method. After the measurement of the 96×96 in-

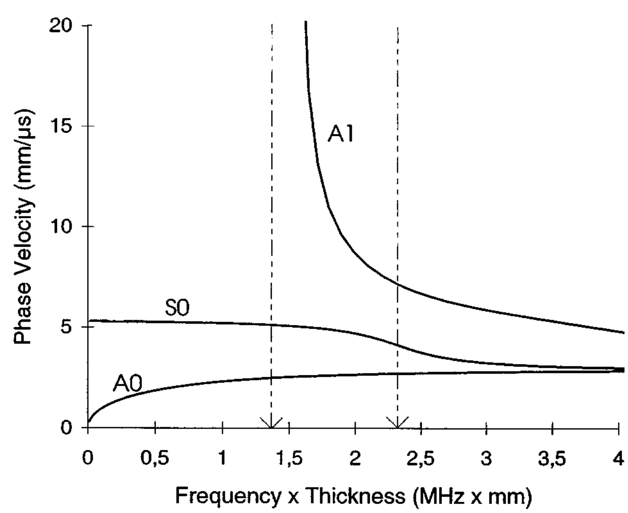

FIG. 7. Dispersion curves of Lamb waves for a steel plate.

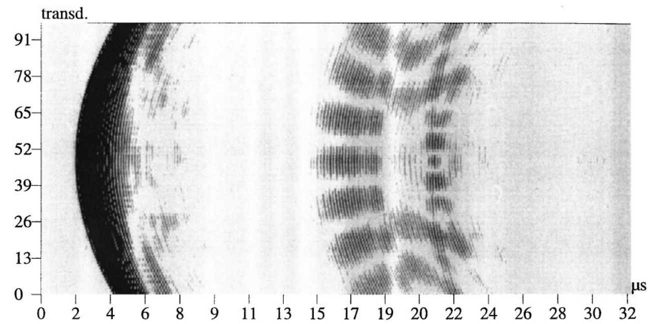

FIG. 8. Echo of the shell received by all the elements of the array after transmission by the central element.

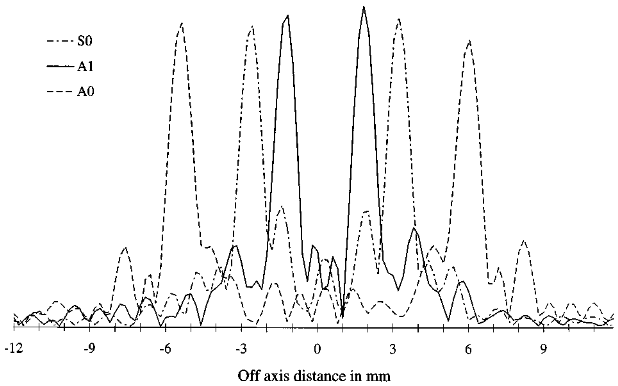

FIG. 10. Pressure patterns obtained by backpropagation of eigenvectors 1, 3, and 5.

terelement impulse responses, the whole process remains numerical. Only the elastic part of the signal is used to calculate the time-reversal operator (between 15 and 25 μs). At 3.05 MHz, the diagonalization of the time-reversal operator has six main eigenvalues. The modulus of the components of each eigenvector (1–6) is represented versus the array element (Fig. 9). The interference fringes are easily observed. They are equivalent at one frequency of the interference pattern observed on the echoes. As in the experiment on two wires (1.4), it means that an eigenvector corresponds to the interference of two coherent point sources.

The numerical backpropagation of each eigenvector allows us to determine the distance between the sources (Fig. 10). Each pair of sources corresponds to one particular Lamb wave. At this frequency, the first and second eigenvectors are associated to the wave S_0, the third and fourth to the wave A_1, and the fifth and sixth to the wave A_0.

The same calculation is done at several frequencies from 2.2 to 4 MHz so that the dispersion curves for the three waves can be plotted (Fig. 11). These curves are very close to the theoretical curves obtained for a steel plate of thickness 0.6 mm. In particular, the determination of the cutoff frequency of the wave A_1 allows us to tell the thickness of the shell.

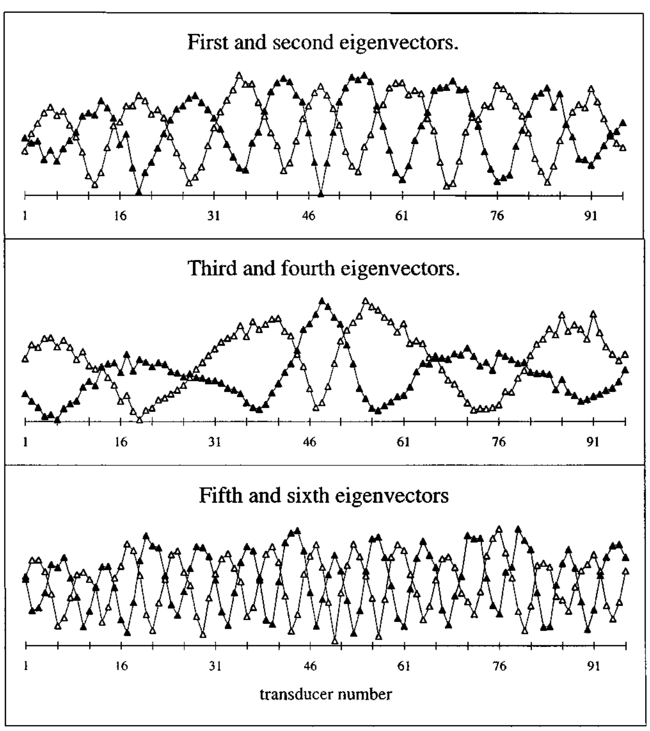

FIG. 9. Modulus of the components of the first six eigenvectors (from top to bottom 1-2, 3-4, 5-6).

3. Resonance frequencies of the shell

The eigenvalue associated to one particular wave depends on the frequency, it is proportional to the level of the contribution of the wave to the scattered field. The generation and reradiation coefficients of the wave are responsible for these fluctuations. Moreover, if the dynamic and duration of the recorded signals allow us to detect several turns of the wave around the shell, a fast modulation of the corresponding eigenvalue is induced, the maxima corresponding to the resonance frequencies of the shell. In the experiment, the wave A_0 is attenuated so fast that only one turn can be observed. But several turns of A_1 and S_0 waves contribute to the scattered field. To take into account these multiple turns the time-reversal operator was calculated using 40 μs of the signal. Then the eigenvalues were calculated from 2.2 to 3.8 MHz. The first six eigenvalues of the time-reversal operator are represented versus frequency (Fig. 12). The two curves

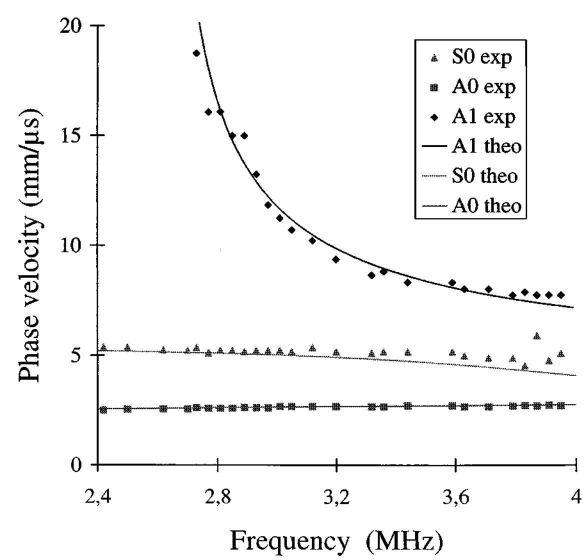

FIG. 11. Dispersion curves: $V_\phi(\omega)$ theory and D.O.R.T. method.

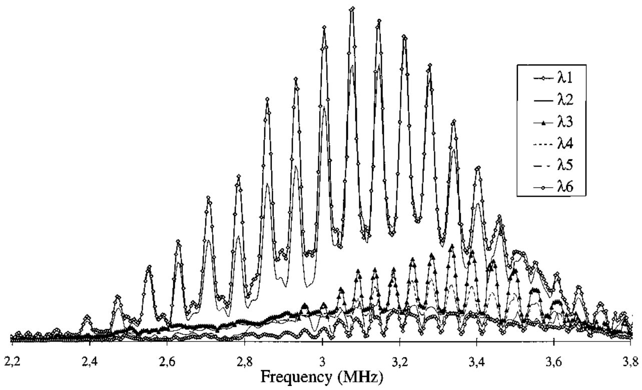

FIG. 12. Eigenvalues of the time-reversal operator obtained from the elastic echo of the shell.

$\lambda_1(\omega)$ and $\lambda_2(\omega)$ correspond to the wave S_0. Their maxima occur at the resonance frequencies of the shell corresponding to this wave. The width of the peaks is mainly due to the length of the recorded signals which allow us to see only three turns of the S_0 wave around the shell. Similar observations can be done for the wave A_1. This wave is associated to the eigenvalues λ_3 and λ_4 around its resonance frequencies and to λ_5 and λ_6 near its antiresonance frequencies. The resonnance peaks are well defined although the contribution of the wave A_1 is weaker than the one of S_0.

To confirm this interpretation, the first eigenvalue $\lambda_1(\omega)$ is compared to the spectrum of the S_0 wave. This last spectrum was obtained by a time-reversal operation as described in Ref. 7. A signal of 40 μs was used to perform the FFT so that only three turns around the shell could be seen. The agreement between the two curves is good (Fig. 13).

4. Limits

One limitation of the D.O.R.T. method is that the generation points of the circumferential waves need to be resolved in space. In the case of two waves of close phase velocities, the separation may not be possible. This explains partly the reason why the velocity of the A_1 wave could not be measured closer to the cutoff frequency. As the phase velocity increases, the two generation points get closer and are no more resolved by the system.

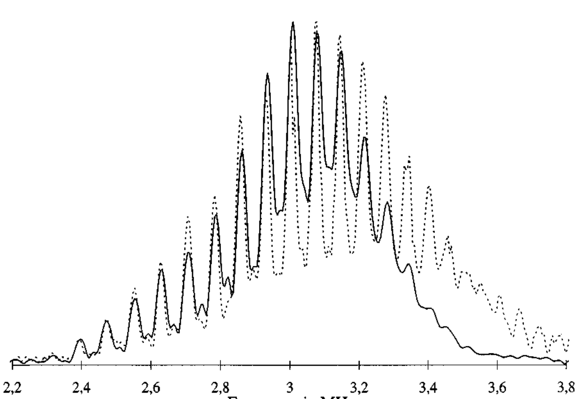

FIG. 13. First eigenvalues of the time-reversal operator (dot line) and experimental spectrum of the $S0$ wave (solid line).

Another limit that we encountered was due to the electronics of the system. The programmable generators we used were built for the time-reversal mirror and could only deliver 20 V peak to peak on each channel. To measure the interelement responses, a more classical generator should be used to get up to 100 V on one channel. This would provide a much better signal-to-noise ratio. Together with the measurement of longer signals, it should allow a more precise determination of the resonance frequencies of the shell.

III. CONCLUSION

The D.O.R.T. method is a new approach to inverse scattering. It allows one to sort various waves contributing to the scattered field and provides information that was previously unavailable. The method was applied to an air-filled stainless-steel cylindrical shell. The circumferential pseudo-Lamb waves A_0, S_0, and A_1 were separated. The dispersion curves of these waves were obtained allowing determination of the thickness of the shell. The eigenvalues of the time-reversal operator provided the resonance frequencies associated with S_0 and A_1 waves and close resonance frequencies of the two different modes were distinguished. Better results should be obtained in the future using electronic devices that provide a higher signal-to-noise ratio for the interelement response signals. Furthermore, the generalization of the method to separate transmit and receive arrays of transducers will widen the domain of the application of this method which should now be applied to other types of scattering problems.

ACKNOWLEDGMENT

The authors wish to express their gratitude to Professor Daniel Royer for many helpful discussions.

[1] N. Gespa, ''La diffusion acoustique par des cibles elastiques de forme géometrique simple,'' Théories et Expérience, CEDOCAR, 1986.
[2] H. Uberall, *Acoustic Resonance Scattering* (Gordon and Breach, New York, 1992).
[3] J. L. Izbicki, G. Maze, and J. Ripoche, ''Influence of the free modes of vibration on the acoustic scattering of a circular cylindrical shell,'' J. Acoust. Soc. Am. **80**, 1215–1219 (1986).
[4] M. Talmant and G. Quentin, ''Backscattering of a short ultrasonic pulse from thin cylindrical shells,'' J. Appl. Phys. **63**, 1857–1863 (1988).
[5] P. Rembert, A. Cand, P. Pareige, M. Talmant, G. Quentin, and J. Ripoche, ''The short pulse method of isolation and identification of resonances: Comparison with a quasiharmonic method and application to axisymmetrical scatterers,'' J. Acoust. Soc. Am. **92**, 3271–3277 (1992).
[6] D. Royer, E. Dieulesaint, and Ph. Leclaire, ''Remote sensing of the thickness of hollow cylinder from optical excitation and detection of Lamb waves,'' in Proceedings of the 1989 Ultrasonic Symposium pp. 1163–1166, IEEE Cat. 89CH2791-2.
[7] J. L. Thomas, P. Roux, and M. Fink, ''Inverse scattering analysis with an acoustic time-reversal mirror,'' Phys. Rev. Lett. **72**, 637–640 (1994).
[8] C. Prada, J. L. Thomas, P. Roux, and M. Fink, ''Acoustic time-reversal and inverse scattering,'' in *Proccedings of the Second International Symposium on Inverse Problems*, edited by H. D. Bui, M. Tanaka, *et al.* (A. A. Balkema, Rotterdam, 1994), pp. 309–316.
[9] C. Prada, F. Wu, and M. Fink, ''The iterative time-reversal mirror: A solution to self-focusing in the pulse echo mode,'' J. Acoust. Soc. Am. **90**, 1119–1129 (1991).
[10] C. Prada, ''Retournement temporel des ondes ultrasonores: Application à la focalisation,'' Thèse de doctorat de l'Université Paris VII, 1991.
[11] M. Fink, ''Time reversal of ultrasonic fields-Part I: Basic principles,'' IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 555–566 (1992).

[12] F. Wu, J. L. Thomas, and M. Fink, ''Time reversal of ultrasonic fields-Part II: Experimental results,'' IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 567–578 (1992).
[13] D. Cassereau and M. Fink, ''Time-reversal of ultrasonic fields-Part III: Theory of the closed time-reversal cavity,'' IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 579–592 (1992).
[14] M. Fink, ''Time-reversal mirrors,'' J. Phys. D **26**, 1333–1350 (1993).
[15] M. Fink, ''Time-reversed acoustics,'' Phys. Today **50**(3), 34–40 (March 1997).
[16] C. Prada, J. L. Thomas, and M. Fink, ''The iterative time-reversal process: analysis of the convergence,'' J. Acoust. Soc. Am. **97**, 62–71 (1995).
[17] G. Bienvenu and L. Kopp, ''Optimality of high resolution array processing using the eigensystem approach,'' IEEE Trans. Acoust., Speech, and Signal Process. **31**, 1235–1247 (1983).
[18] R. O. Schmidt, ''Multiple emitter location and signal parameter estimation,'' IEEE Trans. Antennas Propag. **AP-34**, 276–281 (1986).
[19] C. Prada and M. Fink, ''Eigenmode of the time-reversal operator: a solution to selective focusing in multiple-target media,'' Wave Motion **20**, 151–163 (1994).
[20] C. Prada, S. Manneville, D. Spoliansky, and M. Fink, ''Decomposition of the time reversal operator: Application to detection and selective focusing on two scatterers,'' J. Acoust. Soc. Am. **99**, 2067–2076 (1996).